2011 4th IEEE International Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP)

Efficient and Scalable Demand Response for the Smart Power Grid†

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***Abstract*—A demand response setup is considered entailing a set of appliances with deferrable and non-interruptible tasks. A mixed-integer linear programming model for scheduling the operational periods and power levels of the appliances is formulated in response to known dynamic pricing information with the objective of minimizing the total electricity cost and consumer dissatisfaction. A scalable algorithm yielding a near-optimal solution is developed by enforcing a separable structure, and using Lagrangian relaxation. Thus, the original problem is decomposed to per-appliance subproblems, which can be solved exactly based on dynamic programming. The proximal bundle method is employed to obtain a solution to the convexified version, which helps recovery of a primal feasible solution. Numerical tests validate the proposed approach.**

I. Introduction

Demand response (DR) is a key component of the smart grid, allowing both the consumers and the utilities to benefit through intelligent resource scheduling that capitalizes on the smart grid infrastructure [8]. The electricity prices can be announced via communication networks to the smart meters installed at consumer premises, where the energy consumption can be planned to minimize the total electricity bill, while meeting task-specific operational requirements. By adjusting the prices dynamically, utilities can elicit desired usage patterns from the consumers, saving costs related to power generation, transmission and storage.

To best capture the DR gains, the energy pricing varies depending on the time of use, as well as the aggregate power levels. In this work, the pricing for the entire scheduling horizon is assumed to be known prior to computing the optimal schedule [9]. Based on pricing information, the DR algorithm must determine the operational periods as well as the power consumption of the appliances to minimize the overall electricity cost and maximize user satisfaction.

Appliances may have diverse usage profiles and operational constraints. Some tasks are non-deferrable, while others may be delay-tolerant [14]. Some appliances must expend a given amount of energy to accomplish a task (e.g., charging of an electric vehicle), while others incur varying degrees of satisfaction depending on the energy consumed, but do not have fixed targets (e.g, HVAC systems). Some tasks must be run continuously, while others are interruptible.

Certain combinations of the aforementioned task attributes lead to convex optimization formulations [15]. Most generally, the operation conditions can be modeled using mixed-integer constraints. Our focus is on the latter case. Specifically, the class of appliances considered are deferrable, yet they have set deadlines, and cannot be interrupted. However, the formulation and solution methodologies can be readily extended to handle diverse combinations of attributes.

A significant challenge in DR scheduling is scalability. As the number of appliances that are scheduled jointly increases, the complexity of solving the mixed-integer programs grows rapidly. Therefore, one is forced either to approximate solutions, or to only consider small-scale instances of the problem. In this work, both efficiency and scalability are pursued. The idea is to devise a separable formulation and rely on the Lagrange relaxation technique. The technique was successfully applied to unit commitment problems in power engineering [3], [7], as well as to resource allocation problems for communication networks [12].

The rest of the paper is organized as follows. Section II describes the mathematical model and formulates the optimization problem for DR. Section III discusses the solution approach based on the Lagrangian dual. The results of numerical tests are reported in Section IV, and conclusions are provided in Section V.

II. Problem Formulation

Consider the DR problem of scheduling a set of A appliances over a time horizon of T units, in response to pricing information of electricity that is announced beforehand. The electricity consumer desires to operate appliance $a \in \mathcal{A} \triangleq \{1, 2, \ldots, A\}$ between times α_a and β_a, where $\alpha_a, \beta_a \in \{1, 2, \ldots, T\}$. The tasks are assumed to be deferrable; that is, the appliance may start operation on or after α_a. However, the task has to be completed by time β_a. It is also assumed that once an appliance is turned on, it is not interruptible; that is, the appliance cannot be turned off until the task is completed.

Let p_a^t denote the power expended by appliance a during the t-th time slot. Completing the task by β_a amounts to expending the total required energy $E_{\text{tot},a}$ by that time; i.e., $\sum_{t=1}^{\beta_a} p_a^t \geq E_{\text{tot},a}$. When an appliance is operating, there are lower and upper limits on the power that can be expended, denoted by $\underline{P}_a$ and $\overline{P}_a$, respectively, where $0 \leq \underline{P}_a \leq \overline{P}_a$. Moreover, at each time t, the total load is capped by $P_{\max}$; i.e., $\sum_{a \in \mathcal{A}} p_a^t \leq P_{\max}$.

The electricity cost in time t is captured by an increasing, piecewise linear, convex function $C^t(p)$, which is assumed to be zero at $p = 0$. Thus, $C^t(p)$ can be expressed using constants $c^t(i) \geq 0$ and $d^t(i) \leq 0$, $i = 1, \ldots, I_{C^t}$, as

$$C^t(p) = \max_i \{c^t(i)p + d^t(i)\}, \quad p \geq 0. \tag{1}$$

Often, a simple two-piece function is used to capture the effect of price increase when the total load is higher than a certain threshold [13]. There are computational reasons for advocating piecewise linear convex costs as well. First, the problem can then be decomposed into different appliances using dual decomposition. Secondly, it is feasible to solve the resulting per-appliance optimization problem exactly via dynamic programming (DP) [4].

We also model user dissatisfaction due to delayed completion of tasks, via increasing, piecewise linear, convex functions $D_a^t(E)$, which indicate the degree of dissatisfaction when an amount E of energy remains to be scheduled for appliance a by time $t \leq \beta_a$. It is assumed that $D_a^t(E) = 0$ when $E \leq 0$. Therefore, $D_a^t(E)$ can be

†This work was supported by NSF grants CCF-0830480, 1016605, and ECCS-0824007, 1002180.

expressed as

$$D_a^t(E) = \begin{cases} \max_i\{e_a^t(i)E + f_a^t(i)\}, & E \geq 0 \\ 0, & E < 0 \end{cases} \tag{2}$$

where $e_a^t(i) \geq 0$ and $f_a^t(i) \leq 0$, $i = 1, \ldots, I_{D_a^t}$ are given constants. The demand scheduler determines the times to switch on the individual appliances, as well as potentially time-varying power levels during the operation, in order to minimize the electricity cost and user dissatisfaction.

To formulate the problem, let us introduce binary variables $u_a^t \in \{1, 0\}$ and $x_a^t \in \{1, 0\}$, where $u_a^t = 1$ denotes that appliance a is operational at time t, and x_a^t denotes that appliance a was operational at time $t-1$. Also, the state variable E_a^t denotes the amount of energy expended by appliance a by the beginning of the t-th slot. The relevant optimization problem can then be formulated as

$$\min_{\substack{u_a^t \in \{1,0\}, p_a^t \\ a \in \mathcal{A}, t=1,\ldots,T}} \sum_{t=1}^{T} C^t\left(\sum_{a \in \mathcal{A}} p_a^t\right) + \sum_{t=1}^{T}\sum_{a \in \mathcal{A}} D_a^t\left(E_{\text{tot},a} - E_a^t\right) \tag{3a}$$

subject to:

$$x_a^{t+1} = u_a^t, \quad a \in \mathcal{A},\ t = 1, \ldots, T-1 \tag{3b}$$
$$E_a^{t+1} = E_a^t + p_a^t, \quad a \in \mathcal{A},\ t = 1, \ldots, T-1 \tag{3c}$$
$$E_a^{\beta_a} + p_a^{\beta_a} \geq E_{\text{tot},a}, \quad a \in \mathcal{A} \tag{3d}$$
$$u_a^t = 0 \text{ if } t < \alpha_a \text{ or } \ t > \beta_a, \quad a \in \mathcal{A} \tag{3e}$$
$$u_a^t = 1 \text{ if } E_a^t < E_{\text{tot},a} \text{ and } x_a^t = 1, \quad a \in \mathcal{A},\ t = 1, \ldots, T \tag{3f}$$
$$u_a^t = 0 \text{ if } E_a^t \geq E_{\text{tot},a}, \quad a \in \mathcal{A},\ t = 1, \ldots, T \tag{3g}$$
$$p_a^t \in [\underline{P}_a, \overline{P}_a] \text{ if } u_a^t = 1, \quad a \in \mathcal{A},\ t = 1, \ldots, T \tag{3h}$$
$$p_a^t = 0 \text{ if } u_a^t = 0, \quad a \in \mathcal{A},\ t = 1, \ldots, T \tag{3i}$$
$$\sum_{a \in \mathcal{A}} p_a^t \leq P_{\max}, \quad t = 1, \ldots, T \tag{3j}$$

with the initial conditions $E_a^1 = 0$ and $x_a^1 = 0$, $a \in \mathcal{A}$.

The problem can be reformulated as a mixed-integer linear program, and can be solved using DP. However, since it is coupled across different appliances through the cost functions $C^t(\cdot)$ in (3a), as well as through constraints (3j), it cannot be solved separately per individual appliance. Thus, the complexity grows rapidly as the number of appliances increases, which poses a major hurdle to the scalability of DR. In fact, scheduling non-interruptible tasks is related to the bin packing problem, and solving it exactly generally incurs exponential complexity [11]. In what follows, a separable reformulation that can be solved near-optimally in the dual domain is pursued.

III. Lagrange Relaxation Approach

When an optimization problem has separable structure, the dual method can be applied to decompose the problem into smaller subproblems that can be solved independently, coordinated by the dual variables. Moreover, the duality gap tends to diminish (relative to the optimal objective) as the number of separable terms increases, enabling the dual method to obtain near-optimal solutions. Such an approach has been applied successfully to a number of problems including the unit commitment problem for the power systems [3], [7], as well as resource allocation problems in communication networks [12].

To enforce separability in (3), the piecewise linear convex cost function $C^t(\cdot)$ in (3a) is rewritten. By introducing auxiliary variables v_a^t for $a \in \mathcal{A}$ and $t = 1, \ldots, T$, and referring to (1), one can show that minimizing $\sum_{t=1}^{T} C^t(\sum_{a \in \mathcal{A}} p_a^t)$ is equivalent to minimizing $\sum_{t=1}^{T}\sum_{a \in \mathcal{A}} v_a^t$, under the additional constraints

$$c^t(i)\sum_{a \in \mathcal{A}} p_a^t + d^t(i) \leq \sum_{a \in \mathcal{A}} v_a^t,\ i = 1, \ldots, I_{C^t},\ t = 1, \ldots, T. \tag{4}$$

Also, it is noted that the logical constraints in (3) can be transformed to linear ones involving integer variables [16, Ch. 9]. Specifically, constraints (3h)–(3i) can be equivalently written as

$$\underline{P}_a u_a^t \leq p_a^t \leq \overline{P}_a u_a^t, \quad a \in \mathcal{A},\ t = 1, \ldots, T. \tag{5}$$

Likewise, (3f) and (3g) can be expressed as

$$-E_a^t + (E_{\text{tot},a} + 1 - \epsilon)x_a^t - E_{\text{tot},a}u_a^t - 1 + \epsilon \leq 0 \tag{6}$$
$$E_a^t + (\overline{P}_a + \epsilon)u_a^t - \overline{P}_a - E_{\text{tot},a} \leq 0 \tag{7}$$

respectively, where ϵ is a small positive number (usually the smallest representable by the machine precision).

Overall, (3) is equivalent to

$$\min_{\substack{u_a^t \in \{1,0\}, p_a^t, v_a^t \\ a \in \mathcal{A}, t=1,\ldots,T}} \sum_{t=1}^{T}\sum_{a \in \mathcal{A}} \left[v_a^t + D_a^t(E_{\text{tot},a} - E_a^t)\right] \tag{8a}$$

subject to:

$$c^t(i)\sum_{a \in \mathcal{A}} p_a^t + d^t(i) \leq \sum_{a \in \mathcal{A}} v_a^t, \quad i = 1, \ldots, I_{C^t},\ t = 1, \ldots, T \tag{8b}$$
$$\sum_{a \in \mathcal{A}} p_a^t \leq P_{\max}, \quad t = 1, \ldots, T \tag{8c}$$
$$x_a^{t+1} = u_a^t, \quad a \in \mathcal{A},\ t = 1, \ldots, T-1 \tag{8d}$$
$$E_a^{t+1} = E_a^t + p_a^t, \quad a \in \mathcal{A},\ t = 1, \ldots, T-1 \tag{8e}$$
$$-E_a^t + (E_{\text{tot},a} + 1 - \epsilon)x_a^t - E_{\text{tot},a}u_a^t - 1 + \epsilon \leq 0, \quad a \in \mathcal{A},\ t = 1, \ldots, T \tag{8f}$$
$$E_a^t + (\overline{P}_a + \epsilon)u_a^t - \overline{P}_a - E_{\text{tot},a} \leq 0,\ a \in \mathcal{A},\ t = 1, \ldots, T \tag{8g}$$
$$\underline{P}_a u_a^t \leq p_a^t \leq \overline{P}_a u_a^t, \quad a \in \mathcal{A},\ t = 1, \ldots, T \tag{8h}$$
$$E_a^{\beta_a} + p_a^{\beta_a} \geq E_{\text{tot},a}, \quad a \in \mathcal{A} \tag{8i}$$
$$u_a^t = 0 \text{ if } t < \alpha_a \text{ or } t > \beta_a, \quad a \in \mathcal{A} \tag{8j}$$

The objective function in (8a) is clearly separable into individual appliances. The same holds for constraints (8b) and (8c). All other constraints are trivially separable in a, as they involve only a single appliance per constraint. Therefore, under mild conditions, the duality gap of (P2) approaches zero (relative to the magnitude of the optimal objective) as the number A of appliances grows large [2, Sec. 5.6]. Thus, employing the dual method can yield a near-optimal solution asymptotically as more and more appliances are jointly scheduled.

Introduce Lagrange multipliers $\boldsymbol{\lambda} \triangleq \{\lambda^t(i)\}$ and $\boldsymbol{\mu} \triangleq \{\mu^t\}$ to write the (partial) Lagrangian as

$$\begin{aligned}&\mathcal{L}(\{u_a^t\}, \{p_a^t\}, \{v_a^t\}, \boldsymbol{\lambda}, \boldsymbol{\mu}) \\ &= \sum_{t=1}^{T}\Bigg\{\sum_{a \in \mathcal{A}}\left[v_a^t + D_a^t(E_{\text{tot},a} - E_a^t)\right] \\ &+ \sum_{i=1}^{I_{C^t}} \lambda^t(i)\left(c^t(i)\sum_{a \in \mathcal{A}} p_a^t + d^t(i) - \sum_{a \in \mathcal{A}} v_a^t\right) \\ &+ \mu^t\left(\sum_{a \in \mathcal{A}} p_a^t - P_{\max}\right)\Bigg\}\end{aligned} \tag{9}$$

Then the dual function is given by

$$\mathcal{D}(\boldsymbol{\lambda},\boldsymbol{\mu}) = \inf_{\substack{u_a^t\in\{1,0\},p_a^t,v_a^t,\\ a\in\mathcal{A},t=1,\dots,T}} \mathcal{L}(\{u_a^t\},\{p_a^t\},\{v_a^t\},\boldsymbol{\lambda},\boldsymbol{\mu}) \tag{10}$$

subject to (8d)–(8j)

$$= \sum_{a\in\mathcal{A}} \mathcal{D}_a(\boldsymbol{\lambda},\boldsymbol{\mu}) + \sum_{t=1}^{T}\left(\sum_{i=1}^{I_{Ct}} \lambda^t(i)d^t(i) - \mu^t P_{\max}\right) \tag{11}$$

where

$$\mathcal{D}_a(\boldsymbol{\lambda},\boldsymbol{\mu}) = \inf_{\substack{u_a^t\in\{1,0\},p_a^t,v_a^t,\\ t=1,\dots,T}} \sum_{t=1}^{T}\left[\left(1-\sum_{i=1}^{I_{Ct}}\lambda^t(i)\right)v_a^t + D_a^t(E_{\text{tot},a}-E_a^t) + \left(\mu^t + \sum_{i=1}^{I_{Ct}}\lambda^t(i)c^t(i)\right)p_a^t\right] \tag{12}$$

subject to (8d)–(8j) [with "$a\in\mathcal{A}$" omitted]

Note that $\mathcal{D}_a(\boldsymbol{\lambda},\boldsymbol{\mu})$ is equivalent to

$$\mathcal{D}_a(\boldsymbol{\lambda},\boldsymbol{\mu}) = \begin{cases} \min\limits_{\substack{u_a^t\in\{1,0\},p_a^t,\\ t=1,\dots,T}} \sum\limits_{t=1}^{T}\left[D_a^t(E_{\text{tot},a}-E_a^t) + \left(\mu^t + \sum\limits_{i=1}^{I_{Ct}}\lambda^t(i)c^t(i)\right)p_a^t\right], & \text{if } \sum\limits_{i=1}^{I_{Ct}}\lambda^t(i)=1\ \forall t \\ \text{subject to (8d)–(8j) [with “}a\in\mathcal{A}\text{” omitted]} & \\ -\infty, & \text{otherwise} \end{cases} \tag{13}$$

where the auxiliary variables $\{v_a^t\}$ have now disappeared. The dual problem is given by

$$\max_{\boldsymbol{\lambda}\succeq\mathbf{0},\boldsymbol{\mu}\succeq\mathbf{0}} \mathcal{D}(\boldsymbol{\lambda},\boldsymbol{\mu}) \tag{14a}$$

$$\text{subject to } \sum_{i=1}^{I_{Ct}}\lambda^t(i)=1, \quad t=1,2,\dots,T. \tag{14b}$$

It is noted that given $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ that are feasible for (14), $\mathcal{D}(\boldsymbol{\lambda},\boldsymbol{\mu})$ can be evaluated by solving a DP problem in (13) for each appliance $a\in\mathcal{A}$. Problem (13) belongs to the class of optimal control problems for linear hybrid systems, for which efficient solution methods exist [4], [1]. This is the topic of the next subsection.

A. Evaluation of the Dual Function

Function $\mathcal{D}_a(\boldsymbol{\lambda},\boldsymbol{\mu})$ in (13) can be evaluated using DP for given values of $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$. Define $\gamma^t \triangleq \mu^t + \sum_{i=1}^{I_{Ct}}\lambda^t(i)c^t(i)$ and $\boldsymbol{\gamma}\triangleq(\gamma^1,\dots,\gamma^T)$. With a slight abuse of notation, and after introducing auxiliary variables $\{w_a^t\}$, $\mathcal{D}_a$ can be rewritten as [cf. (2)]

$$\mathcal{D}_a(\boldsymbol{\gamma}) = \min_{\substack{u_a^t\in\{1,0\},p_a^t,w_a^t\\ t=1,\dots,T}} \sum_{t=1}^{T}\left[w_a^t + \gamma^t p_a^t\right] \tag{15a}$$

subject to (8d)–(8j) [with "$a\in\mathcal{A}$" omitted] (15b)

$$w_a^t \ge 0, \quad t=1,\dots,T \tag{15c}$$

$$e_a^t(i)(E_{\text{tot},a}-E_a^t) + f_a^t(i) \le w_a^t, \quad i=1,\dots,I_{D_a^t},\ t=1,\dots,T \tag{15d}$$

The backward recursion for solving DP is given through the value functions $J^t(x_a^t,E_a^t;\boldsymbol{\gamma})$ as

$$J^T(x_a^T,E_a^T;\boldsymbol{\gamma}) = \min_{u_a^T\in\{1,0\},p_a^T,w_a^T} w_a^T + \gamma^T p_a^T \tag{16}$$

subject to: (8f)–(8j) and (15c)–(15d) [with t set to T, and "$a\in\mathcal{A}$" and "$t=1,\dots,T$" omitted]

$$J^t(x_a^t,E_a^t;\boldsymbol{\gamma}) = \min_{\substack{u_a^t\in\{1,0\},\\ p_a^t,w_a^t}} w_a^t + \gamma^t p_a^t + J^{t+1}(u_a^t, E_a^t + p_a^t;\boldsymbol{\gamma})$$

subject to: (8f)–(8j) and (15c)–(15d) [with "$a\in\mathcal{A}$" and "$t=1,\dots,T$" omitted], for $t=T-1,\dots,1$. (17)

The optimal objective $D_a(\boldsymbol{\gamma})$ is then equal to $J^1(0,0;\boldsymbol{\gamma})$. Evaluating $J^t(\cdot)$ requires solving a multiparametric mixed-integer linear program, with the parameters being the state variables x_a^t and E_a^t. It can be shown that the solutions to (16)–(17), as well as the value functions, are piecewise linear functions defined on polyhedral partitions of the feasible state space [4]. Therefore, the backward induction can be performed exactly without discretizing the state or the input spaces.

The computational complexity may be still high, when (13) must be solved repeatedly for different values of $\boldsymbol{\gamma}$, as is the case of using an iterative algorithm to obtain the optimal dual variables. When the scheduling horizon is short, one can mitigate this difficulty by solving (13) with $\boldsymbol{\gamma}$ regarded as parameters as well. In general, this involves a multiparametric mixed-integer *quadratic* program due to the bilinear terms $\gamma^t p_a^t$ in (16)–(17). Both the one-shot mixed-integer programming and the DP approaches are feasible for parametric solution. In the numerical tests in Sec. IV, the former approach is taken. Once the parametric solutions are obtained, evaluating $\mathcal{D}_a(\boldsymbol{\gamma})$ amounts to simple function evaluations. This strategy is especially effective in reducing complexity when one desires to schedule multiple instances of tasks with identical requirements.

B. Solution to the Dual Problem (14)

Since the Lagrangian in (9) is nonconvex, the convex dual function $\mathcal{D}(\boldsymbol{\lambda},\boldsymbol{\mu})$ is nondifferentiable. Optimization algorithms suitable for nondifferentiable objectives include the subgradient method or the ellipsoid method [5], which yield optimal dual variables. However, due to the nonzero duality gap, the primal solution recovered as an optimizer of (13) at the dual optimal solution does not necessarily satisfy the constraints of the original problem. In [3], the exponential method of multipliers was employed to solve not only the dual of a unit commitment problem, but also the "dual-to-dual" as a byproduct. That is, the preferred method is the one that provides not only the optimal dual variables, but also the solution to the convexified primal problem, which can help obtain a primal feasible solution through a heuristic rule. Since the duality gap diminishes in the number of separable units, the so-obtained primal feasible solution is close to the optimal solution when a large-scale problem is solved. However, the method in [3] requires enumeration of all breakpoints of the piecewise linear objective, which is not straightforward for the problem at hand.

An alternative approach is to use the proximal bundle method [10], [6]. The method can again handle nondifferentiable objectives and yields solutions to the convexified problem at no additional cost. Specifically, consider a separable problem of the form

$$\min_{\{\mathbf{z}_a\in Z_a\}} \sum_{a=1}^{A} g_a(\mathbf{z}_a) \text{ s.t. } \sum_{a=1}^{A} h_{ka}(\mathbf{z}_a)\le 0,\ k=1,\dots,K. \tag{18}$$

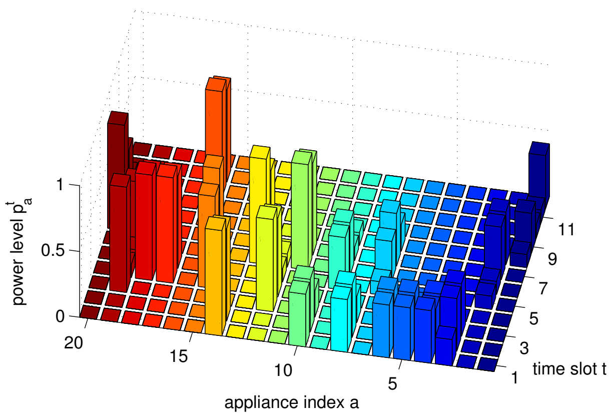

Fig. 1. Computed power profile.

Let $\mathbf{h}_a \triangleq (h_{1a}, \ldots, h_{Ka})$. Then, based on the Lagrangian dual, the method in [6] yields the solution to the convexified problem

$$\min_{\{\nu_{aj}\geq 0\},\{\mathbf{z}_a(j)\in Z_a\}} \sum_{a=1}^{A}\sum_{j=1}^{K+1} \nu_{aj} g_a(\mathbf{z}_a(j)) \tag{19a}$$

$$\text{subject to } \sum_{a=1}^{A}\sum_{j=1}^{K+1} \nu_{aj}\mathbf{h}_a(\mathbf{z}_a(j)) \preceq \mathbf{0}, \quad \sum_{j=1}^{K+1} \nu_{aj} = 1,\ \forall a \tag{19b}$$

If one views the variable ν_{aj} as the probability of operating unit a using "mode" $\mathbf{z}_a^j$, the solution to (19) is seen to minimize the expectation of the original objective while satisfying the constraints in the average sense. Moreover, it turns out that the number of units with fractional ν_{aj} is at most K [6], [2], and oftentimes smaller in practice. Therefore, for large-scale problems in which $A \gg K$, the solutions for only a few units need to be perturbed to obtain a feasible solution to the original problem (18).

C. Recovery of a Primal Feasible Solution

It is noted that once the on/off profile $\{u_a^t\}$ of all appliances at each time step is determined, the optimal power consumption profile $\{p_a^t\}$ can be obtained simply by solving a linear program. As was discussed in Sec. III-B, the proximal bundle method yields the set of $\{u_a^t(j)\}$ and $\{p_a^t(j)\}$ along with the fractions $\{\nu_{aj}\}$, which solve the relaxed version of the DR problem per (19). Since most of the appliances obtain a single $u_a^t(j)$ with $\nu_{aj} = 1$, the simple heuristic of choosing $u_a^t(j)$ corresponding to the largest ν_{aj} value for each appliance a very often obtains a feasible solution. Should the subsequent linear program for the powers turn out to be infeasible, the next candidate on/off profile is tried in the order of decreasing $\prod_{a\in\mathcal{A}} \nu_{aj}$.

IV. Numerical Test

To verify the proposed algorithm, a preliminary numerical test is performed. A total of $A = 20$ appliances are scheduled over a horizon of length $T = 12$. The values of $E_{\text{tot},a}$ and $\overline{P}_a$ for the first 10 appliances are set to 1 and 0.4, respectively, and for the rest to 1.5 and 0.8, while $\underline{P}_a = 0.1$ for all $a \in \mathcal{A}$. The appliances can start operation any time ($\alpha_a = 1,\ \forall a$), but must finish the tasks by the end of the horizon ($\beta_a = T,\ \forall a$). A two-piece piecewise linear convex cost function $C^t(p)$ is employed for all t, with slope 1 when $0 \leq p \leq 2$, and slope 5 for $p > 2$. The disutility functions $D_a^t(E)$ are identical for all a and t, with $D_a^t(E) = 0$ when $E < 0$, and $D_a^t(E) = E$ when $E \geq 0$. The proximal bundle method returned the optimal solution in 15 iterations, and 9 out of 20 appliances received fractional values of ν_{aj}. The most "probable" candidate resulted in the feasible solution depicted in Fig. 1. The optimal objective turned out to be 223.7, slightly larger than the optimal dual objective of 222, signifying that the obtained solution is near-optimal.

V. Conclusions

A DR algorithm that determines the operation schedules of a set of appliances given electricity prices and various task constraints was developed. For specificity, a class of deferrable yet non-interruptible appliances was considered with potentially time-varying electricity and user dissatisfaction costs. A mixed-integer linear programming formulation was obtained by modeling the costs as piecewise linear convex. To alleviate high complexity of exactly solving the problem, the dual method was employed to decompose the problem into per-appliance subproblems, which can be solved exactly using DP without discretizing the search space. As the duality gap diminishes in the number of appliances, a near-optimal solution can be found. The proximal bundle method for solving the dual problem yielded the solutions to the convexified problem, which were valuable for finding the primal feasible solutions. The numerical tests verified that indeed a near-optimal DR schedule can be obtained.

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