UMBC
Fall 1999

CMSC–441, Section 201
Algorithms

Exam I, October 6

Name _____ SSN _____

Problem 1	/	15
Problem 2	/	20
Problem 3	/	20
Problem 4	/	25
Problem 5	/	20
Total	/	100

Instructions:

1. Write your name and Social Security Number on this handout.
2. Make sure that this handout has **3** pages and **5** problems.
3. Solve each of the following five problems.
4. Write all your answers in a readable form in the blue book.
5. Number all your answers.
6. Justify all the steps in your answers.
7. You must return this handout and your blue book.

(15 points) **Problem 1. Asymptotic Notation**
State whether the following statements are TRUE or FALSE.

(a) $5n + 897 \in \Theta(n^2)$

(b) $5n^2 + 0.5n \in O(n)$

(c) $5n^2 - 5\sqrt{n} \in O(n \log n)$

(d) $2^{n+1} + n^2 \in O(2^n)$

(e) $2^{2n} + 1/n \in O(2^n)$

(20 points) **Problem 2. Substitution Method**
Using the **substitution method** show that the solution for the following recurrence is $\Theta(n)$:

$T(n) = T(n/3) + T(n/2) + 3n/2$

(20 points) **Problem 3. Iteration Method**
Using the **iteration method** find a Θ–bound for the following recurrence:

$T(n) = 2T(n-1) + 1$

(25 points) **Problem 4. Recurrences**
Solve the following recurrences. Use Master theorem where ever possible. If applicable, clearly state which case of the Master theorem applies.

(a) $T(n) = 9T(n/8) + n$

(b) $T(n) = 100T(n/10) + n^2 + 100$

(c) $T(n) = 2T(n/4) + n^3$

(d) $T(n) = 47T(n/7) + 5n^2$

(e) $T(n) = 2T(\sqrt{n}) + 1$

(20 points) **Problem 5.**
Consider the algorithm *Mystery* given below.

(a) Give a recurrence for the worst–case running time of the *Mystery* algorithm.

(b) Find a Θ–bound on the worst–case running time of *Mystery* algorithm by solving the recurrence in (a).

```
Mystery(A, p, r)
(1)    for i ← p to r do
(2)      for j ← p + 1 to r do
(3)        A[i] ← √(A[j] + A[i])
(4)      end–for
(5)    end–for
(6)    n ← r − p + 1
(7)    if n ≤ 1 then
(8)      return
(9)    k ← ⌊n/2⌋
(10)     if (A[p] > A[r] + 3) then
(11)       Mystery(A, p, p + k)
(12)     else
(13)       Mystery(A, p + k + 1, r)
(14)     end–if
```