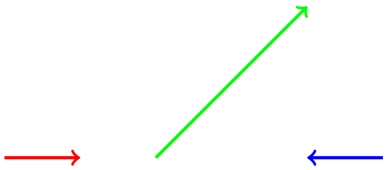
Linear Algebra Review

CMSC 435/634

Abstract Vectors

($\vec{u}$, $\vec{v}$, $\vec{w}$ vectors; a, b, c scalars)



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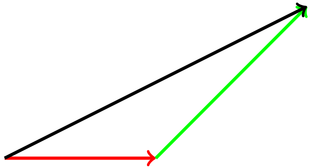

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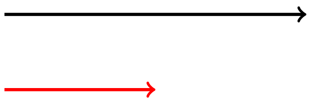

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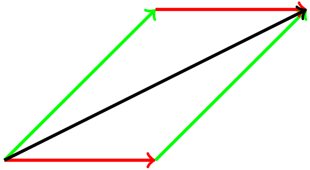

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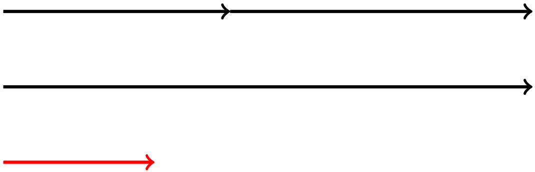

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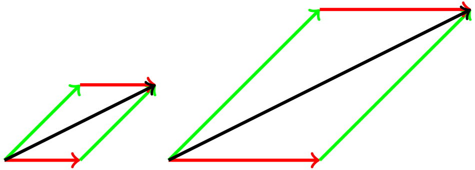

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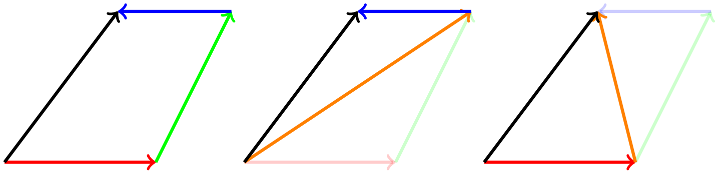
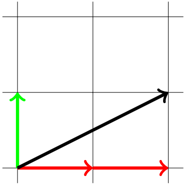


Basis Vectors

Vector as linear combination of *basis vectors*

- $\vec{v} = 2\hat{i} + 1\hat{j} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$



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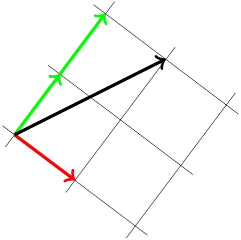

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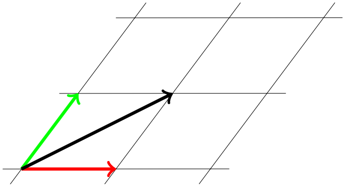

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- Row: $\vec{v} = \begin{bmatrix} v_0 & v_1 \end{bmatrix}$
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- I like columns for points/vectors, rows for normals

Matrices

- Matrix: $A = \begin{bmatrix} a_{0,0} & a_{0,1} \\ a_{1,0} & a_{1,1} \end{bmatrix} = [a_{i,j}] = [a_{row,column}]$

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Matrix Code

- Math: $C = A\ B$
- Components: $c_{i,j} = \sum_\alpha a_{i,\alpha}\ b_{\alpha,j}$
- Code:

```
for(int i=0; i<N; ++i) {
  for(int j=0; j<M; ++j) {
    c[i][j] = 0;
    for(int α=0; α<K; α++) {
      c[i][j] = c[i][j] + a[i][α] * b[α][j];
    }
  }
}
```

Adjugate and Inverse

- Inverse: $A^{-1}A = AA^{-1} = I$
- Determinant: $|A|$
  - $|a| = a$
  - $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a|d| - b|c|$
  - $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$
- Adjugate: $A^* = cof(A)^T$ (matrix of cofactors $cof(A)$)
  - Sometimes called Adjoint or Adjunct
- $A^{-1} = \frac{A^*}{|A|}$

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- Equivalent notations
  - Vector: $\vec{u} \bullet \vec{v}$
  - Matrix: $U^T V$
  - $\sum_\alpha u_\alpha v_\alpha$

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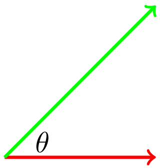

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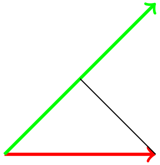

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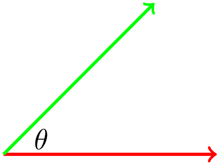
  - If $|\vec{u}| = |\vec{v}| = 1$, gives just $\cos\theta$



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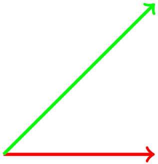
- *Orthonormal*: set of vectors both orthogonal and normal
- *Orthogonal matrix*: rows (& columns) **orthonormal**
  - For orthogonal matrices, $A^{-1} = A^T$

3D Cross Product

$\vec{u} \times \vec{v}$



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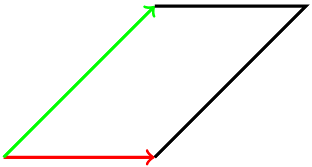

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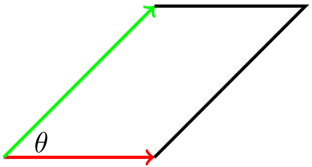

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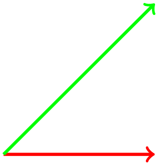
- direction = perpendicular to $\vec{u}$ and $\vec{v}$ (right hand rule)



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- $\vec{w} = \vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & & \\ \hat{j} & U & V \\ \hat{k} & & \end{vmatrix}$

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- $\begin{bmatrix} w_0 \\ w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} u_1 v_2 - u_2 v_1 \\ u_2 v_0 - u_0 v_2 \\ u_0 v_1 - u_1 v_0 \end{bmatrix}$

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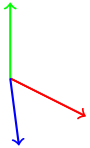
- Positive terms follow 012012 order / negative follow 210210 order

Building an Orthogonal Basis

Vectors $\vec{u}$, $\vec{v}$, $\vec{w}$



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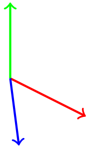

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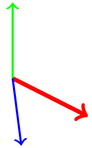
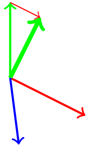
- Gram-Schmidt (any number of dimensions)
  - $\vec{u'} = \vec{u}$



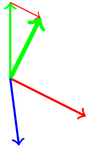
Building an Orthogonal Basis

Vectors $\vec{u}$, $\vec{v}$, $\vec{w}$

- Gram-Schmidt (any number of dimensions)
  - $\vec{u'} = \vec{u}$
  - $\vec{v'} = \vec{v} - \hat{u}' \left(\vec{v} \bullet \hat{u}' \right)$



Building an Orthogonal Basis



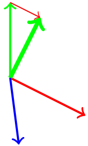
Vectors $\vec{u}$, $\vec{v}$, $\vec{w}$

- Gram-Schmidt (any number of dimensions)
  - $\vec{u'} = \vec{u}$
  - $\vec{v'} = \vec{v} - \frac{\vec{u'}}{|\vec{u'}|}\left(\vec{v} \bullet \frac{\vec{u'}}{|\vec{u'}|}\right)$

Building an Orthogonal Basis

Vectors $\vec{u}$, $\vec{v}$, $\vec{w}$

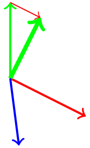
- Gram-Schmidt (any number of dimensions)
  - $\vec{u'} = \vec{u}$
  - $\vec{v'} = \vec{v} - \vec{u'}\ \vec{v} \bullet \vec{u'} / |\vec{u'}|^2$



Building an Orthogonal Basis

Vectors $\vec{u}$, $\vec{v}$, $\vec{w}$

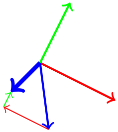
- Gram-Schmidt (any number of dimensions)
  - $\vec{u'} = \vec{u}$
  - $\vec{v'} = \vec{v} - \vec{u'}\ \vec{v} \bullet \vec{u'} / \vec{u'} \bullet \vec{u'}$



Building an Orthogonal Basis

Vectors $\vec{u}$, $\vec{v}$, $\vec{w}$

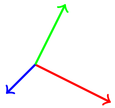
- Gram-Schmidt (any number of dimensions)
  - $\vec{u'} = \vec{u}$
  - $\vec{v'} = \vec{v} - \vec{u'}\ \vec{v} \bullet \vec{u'} / \vec{u'} \bullet \vec{u'}$
  - $\vec{w'} = \vec{w} - \vec{u'}\ \vec{w} \bullet \vec{u'} / \vec{u'} \bullet \vec{u'} - \vec{v'}\ \vec{w} \bullet \vec{v'} / \vec{v'} \bullet \vec{v'}$



Building an Orthogonal Basis

Vectors $\vec{u}$, $\vec{v}$, $\vec{w}$

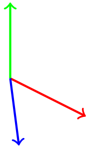
- Gram-Schmidt (any number of dimensions)
  - $\vec{u'} = \vec{u}$
  - $\vec{v'} = \vec{v} - \vec{u'}\ \vec{v} \bullet \vec{u'} / \vec{u'} \bullet \vec{u'}$
  - $\vec{w'} = \vec{w} - \vec{u'}\ \vec{w} \bullet \vec{u'} / \vec{u'} \bullet \vec{u'} - \vec{v'}\ \vec{w} \bullet \vec{v'} / \vec{v'} \bullet \vec{v'}$



Building an Orthogonal Basis

Vectors $\vec{u}$, $\vec{v}$, $\vec{w}$

- Gram-Schmidt (any number of dimensions)
  - $\vec{u'} = \vec{u}$
  - $\vec{v'} = \vec{v} - \vec{u'}\ \vec{v} \bullet \vec{u'} / \vec{u'} \bullet \vec{u'}$
  - $\vec{w'} = \vec{w} - \vec{u'}\ \vec{w} \bullet \vec{u'} / \vec{u'} \bullet \vec{u'} - \vec{v'}\ \vec{w} \bullet \vec{v'} / \vec{v'} \bullet \vec{v'}$
- Cross-product (3D only)



Building an Orthogonal Basis

Vectors $\vec{u}$, $\vec{v}$, $\vec{w}$

- Gram-Schmidt (any number of dimensions)
  - $\vec{u'} = \vec{u}$
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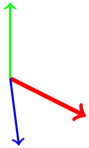
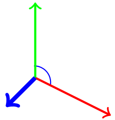
  - $\vec{w'} = \vec{w} - \vec{u'}\ \vec{w} \bullet \vec{u'} / \vec{u'} \bullet \vec{u'} - \vec{v'}\ \vec{w} \bullet \vec{v'} / \vec{v'} \bullet \vec{v'}$
- Cross-product (3D only)
  - $\vec{u'} = \vec{u}$



Building an Orthogonal Basis

Vectors $\vec{u}$, $\vec{v}$, $\vec{w}$

- Gram-Schmidt (any number of dimensions)
  - $\vec{u'} = \vec{u}$
  - $\vec{v'} = \vec{v} - \vec{u'}\ \vec{v} \bullet \vec{u'} / \vec{u'} \bullet \vec{u'}$
  - $\vec{w'} = \vec{w} - \vec{u'}\ \vec{w} \bullet \vec{u'} / \vec{u'} \bullet \vec{u'} - \vec{v'}\ \vec{w} \bullet \vec{v'} / \vec{v'} \bullet \vec{v'}$
- Cross-product (3D only)
  - $\vec{u'} = \vec{u}$
  - $\vec{w'} = \vec{u'} \times \vec{v}$



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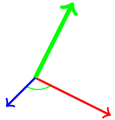
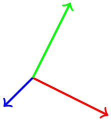
  - $\vec{v'} = \vec{v} - \vec{u'}\ \vec{v} \bullet \vec{u'} / \vec{u'} \bullet \vec{u'}$
  - $\vec{w'} = \vec{w} - \vec{u'}\ \vec{w} \bullet \vec{u'} / \vec{u'} \bullet \vec{u'} - \vec{v'}\ \vec{w} \bullet \vec{v'} / \vec{v'} \bullet \vec{v'}$
- Cross-product (3D only)
  - $\vec{u'} = \vec{u}$
  - $\vec{w'} = \vec{u'} \times \vec{v}$
  - $\vec{v'} = \vec{w'} \times \vec{u'}$



Building an Orthogonal Basis



Vectors $\vec{u}$, $\vec{v}$, $\vec{w}$

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    - $\vec{u'} = \vec{u}$
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    - $\vec{w'} = \vec{w} - \vec{u'}\ \vec{w} \bullet \vec{u'} / \vec{u'} \bullet \vec{u'} - \vec{v'}\ \vec{w} \bullet \vec{v'} / \vec{v'} \bullet \vec{v'}$
- Cross-product (3D only)
    - $\vec{u'} = \vec{u}$
    - $\vec{w'} = \vec{u'} \times \vec{v}$
    - $\vec{v'} = \vec{w'} \times \vec{u'}$