Viewing

CMSC 435/634

Spaces

- Object / Model
  - Logical coordinates for modeling
  - May have several more levels
- World
  - Common coordinates for everything
- View / Camera / Eye
  - eye/camera at (0,0,0), looking down Z (or -Z) axis
  - planes: left, right, top, bottom, near/hither, far/yon
- Normalized Device Coordinates (NDC) / Clip
  - Visible portion of scene from (-1,-1,-1) to (1,1,1)
- Raster / Pixel / Viewport
  - 0,0 to x-resolution, y-resolution
- Device / Screen
  - May translate to fit actual screen

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Model→World / Model→View

- Model→World
  - All shading and rendering in World space
  - Transform all objects and lights
- Ray tracing implicitly does World→Raster
- Model→View
  - Serves just as well for single view

World→View

- Also called Viewing or Camera transform
- LookAt
  - $\overrightarrow{from}, \overrightarrow{to}, \overrightarrow{up}$
  - $\left[\ \vec{u}\ \middle|\ \vec{v}\ \middle|\ \vec{w}\ \middle|\ \overrightarrow{from}\ \right]$
- Roll / Pitch / Yaw
  - Translate to camera center, rotate around camera
  - $R_z\ R_x\ R_y\ T$
  - Can have gimbal lock
- Orbit
  - Rotate around object center, translate out
  - $T\ R_z\ R_x\ R_y$
  - Also can have gimbal lock

View→NDC

- Also called *Projection* transform
- Orthographic / Parallel
  - Translate & Scale to view volume
  - $$\begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
- Perspective
  - More complicated...

NDC→Raster

- Also called *Viewport* transform
- $[-1,1],[-1,1],[-1,1] \rightarrow [-\frac{1}{2}, n_x - \frac{1}{2}], [-\frac{1}{2}, n_y - \frac{1}{2}], [-\frac{1}{2}, n_z - \frac{1}{2}]$
  - Translate to $[0,2],[0,2],[0,2]$
  - Scale to $[0,n_x],[0,n_y],[0,n_z]$
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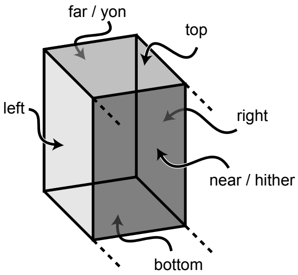
Raster→Screen

- Usually just a translation
  - More complicated for tiled displays, domes, etc.
- Usually handled by windowing system

Perspective View Frustum

- Orthographic view volume is a rectangular volume
- Perspective is a truncated pyramid or *frustum*



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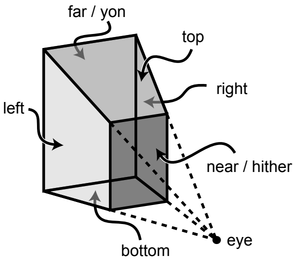

Perspective Transform

- Ray tracing
  - Given screen (s^x, s^y), parameterize all points $\vec{p}$
- Perspective Transform
  - Given $\vec{p}$, find (s^x, s^y)
  - Use similar triangles
  - $s^x/d = p^x/p^z$

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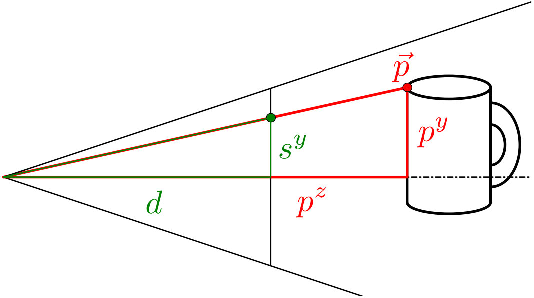

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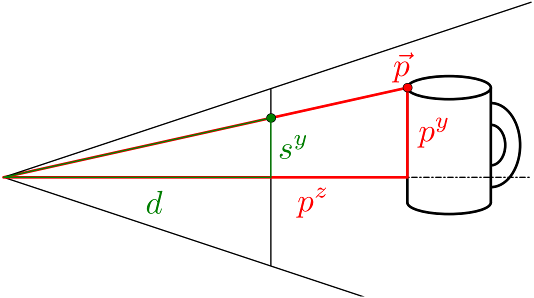

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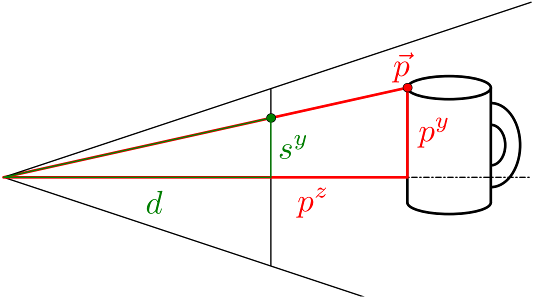

Homogeneous Equations

- Same degree for every term
- Introduce a new redundant variable
- $aX + bY + c = 0$
  - $X = x/w, Y = y/w$
    $a\,x/w + b\,y/w + c = 0$
    $\rightarrow a\,x + b\,y + c\,w = 0$
- $aX^2 + bXY + cY^2 + dX + eY + f = 0$

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Homogeneous Coordinates

- Rather than $(x, y, z, 1)$, use (x, y, z, w)
- Real 3D point is $(X, Y, Z) = (x/w, y/w, z/w)$
- Can represent Perspective Transform as 4x4 matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} p^x \\ p^y \\ p^z \\ 1 \end{bmatrix} = \begin{bmatrix} p^x \\ p^y \\ p^z \\ p^z/d \end{bmatrix} \rightarrow \begin{bmatrix} d\, p^x/p^z \\ d\, p^y/p^z \\ d \end{bmatrix}$$

Homogeneous Depth

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} p^x \\ p^y \\ p^z \\ 1 \end{bmatrix} = \begin{bmatrix} p^x \\ p^y \\ p^z \\ p^z/d \end{bmatrix} \rightarrow \begin{bmatrix} d\, p^x/p^z \\ d\, p^y/p^z \\ d \end{bmatrix}$$

- Lose depth information
- Can't get $d\, p'^z/p^z = p^z$
  - Plus $x/z, y/z, z$ isn't linear
- Use *Projective Geometry*

Projective Geometry

- If x, y, z lie on a plane, $x/z, y/z, 1/z$ also lie on a plane
- $1/z$ is strictly ordered: if $z_1 < z_2$, then $1/z_1 > 1/z_2$
- New matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} p^x \\ p^y \\ p^z \\ 1 \end{bmatrix} = \begin{bmatrix} p^x \\ p^y \\ 1 \\ p^z \end{bmatrix} \rightarrow \begin{bmatrix} p^x/p^z \\ p^y/p^z \\ 1/p^z \end{bmatrix}$$

Getting Fancy

- Add scale & translate
  - Field of view
  - near/far range

$$\begin{bmatrix} a & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & b & c \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} p^x \\ p^y \\ p^z \\ 1 \end{bmatrix} = \begin{bmatrix} a\, p^x \\ a\, p^y \\ b\, p^z + c \\ -p^z \end{bmatrix} \rightarrow \begin{bmatrix} -a \frac{p^x}{p^z} \\ -a \frac{p^y}{p^z} \\ -b - \frac{c}{/} p^z \end{bmatrix}$$

- $a = cotan(fieldOfView/2)$
- Solve for $n \rightarrow -1$ and $f \rightarrow 1$
  - $b = \frac{n+f}{n-f}$
  - $c = \frac{2\,n\,f}{f-n}$

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On Field of View

- Given image dimensions, set distance
  - Camera image sensor and focal length
- Given field of view angle in square window
- Non-square aspect ratio
  - Given horizontal (or vertical) field of view
  - Given diagonal field of view
- Off-center projection
  - Tiled displays
  - Head tracking

OpenGL

- glMatrixMode(GL_MODELVIEW)
  - glTranslatef(x,y,z)
  - glRotatef(degrees,x,y,z)
  - glScalef(x,y,z)
  - gluLookAt(eyeX,eyeY,eyeZ, atX,atY,atZ, upX,upY,upZ)
- glMatrixMode(GL_PERSPECTIVE)
  - glOrtho(nearL,nearR,nearT,nearB, near,far)
  - glFrustum(nearL,nearR,nearT,nearB, near,far)
  - gluPerspective(yFOV,aspect, near,far)
  - glViewport(left,right, width,height)
- raw interface
  - glLoadIdentity()
  - glLoadMatrixf(float*)
  - glMultMatrixf(float*)