

I ran matlab on retriever to replicate the numerical example at the end of the Deerwester paper. The results were a little surprising!

To start matlab, type 'matlab'

Then, I typed in the matrix as shown (removing some of my typos)

```
retriever> matlab
...
>> x = [ 1 0 0 1 0 0 0 0 0;
1 0 1 0 0 0 0 0 0;
1 1 0 0 0 0 0 0 0;
0 1 1 0 1 0 0 0 0;
0 1 1 2 0 0 0 0 0;
0 1 0 0 1 0 0 0 0;
0 1 0 0 1 0 0 0 0;
0 0 1 1 0 0 0 0 0;
0 1 0 0 0 0 0 0 1;
0 0 0 0 0 1 1 1 0;
0 0 0 0 0 1 1 1 1;
0 0 0 0 0 0 1 1 1]
```

and matlab replies

```
x =
 1  0  0  1  0  0  0  0  0
 1  0  1  0  0  0  0  0  0
 1  1  0  0  0  0  0  0  0
 0  1  1  0  1  0  0  0  0
 0  1  1  2  0  0  0  0  0
 0  1  0  0  1  0  0  0  0
 0  1  0  0  1  0  0  0  0
 0  0  1  1  0  0  0  0  0
 0  1  0  0  0  0  0  0  1
 0  0  0  0  0  1  1  1  0
 0  0  0  0  0  0  1  1  1
 0  0  0  0  0  0  0  1  1
```

```
>> help svd
```

SVD Singular value decomposition.

[U,S,V] = SVD(X) produces a diagonal matrix S, of the same dimension as X and with nonnegative diagonal elements in decreasing order, and unitary matrices U and V so that $X = U*S*V'$.

S = SVD(X) returns a vector containing the singular values.

[U,S,V] = SVD(X,0) produces the "economy size" decomposition. If X is m-by-n with $m > n$, then only the first n columns of U are computed and S is n-by-n.

See also SVDS.

Overloaded methods
help sym/svd.m

I used a form of the svd command that only computes a given number of dimensions. The svds form is usually faster than ordinary svd, but it might not be as accurate!

```
>> [u,s,v]=svds(x,9);
>> u
```

u =

Columns 1 through 7

```
-0.2214 -0.1132 -0.2890  0.4148  0.1063  0.3410  0.5227
-0.1976 -0.0721 -0.1350  0.5522 -0.2818 -0.4959 -0.0704
-0.2405  0.0432  0.1644  0.5950  0.1068  0.2550 -0.3022
-0.4036  0.0571  0.3378 -0.0991 -0.3317 -0.3848  0.0029
-0.6445 -0.1673 -0.3611 -0.3335  0.1590  0.2065 -0.1658
-0.2650  0.1072  0.4260 -0.0738 -0.0803  0.1697  0.2829
-0.2650  0.1072  0.4260 -0.0738 -0.0803  0.1697  0.2829
-0.3008 -0.1413 -0.3303 -0.1881 -0.1148 -0.2722  0.0330
-0.2059  0.2736  0.1776  0.0324  0.5372 -0.0809 -0.4669
```

```

-0.0127  0.4902  -0.2311  -0.0248  -0.5942  0.3921  -0.2883
-0.0361  0.6228  -0.2231  -0.0007  0.0683  -0.1149  0.1596
-0.0318  0.4505  -0.1411  0.0087  0.3005  -0.2773  0.3395

```

Columns 8 through 9

```

0.0605  0.4067
0.0099  0.1089
-0.0623 -0.4924
0.0004  -0.0123
-0.0343 -0.2707
0.0161  0.0539
0.0161  0.0539
0.0190  0.1653
0.0363  0.5794
-0.2546 0.2254
0.6811  -0.2320
-0.6784 -0.1825

```

>> s

s =

Columns 1 through 7

```

3.3409    0    0    0    0    0    0
  0  2.5417    0    0    0    0    0
  0    0  2.3539    0    0    0    0
  0    0    0  1.6445    0    0    0
  0    0    0    0  1.5048    0    0
  0    0    0    0    0  1.3064    0
  0    0    0    0    0    0  0.8459
  0    0    0    0    0    0    0
  0    0    0    0    0    0    0

```

Columns 8 through 9

```

0    0
0    0

```

```

0    0
0    0
0    0
0    0
0    0
0.5601    0
0  0.3637

```

>> v

v =

Columns 1 through 7

```

-0.1974 -0.0559 -0.1103  0.9498 -0.0457  0.0766  0.1773
-0.6060  0.1656  0.4973  0.0286  0.2063  0.2565 -0.4330
-0.4629 -0.1273 -0.2076 -0.0416 -0.3783 -0.7244 -0.2369
-0.5421 -0.2318 -0.5699 -0.2677  0.2056  0.3689  0.2648
-0.2795  0.1068  0.5054 -0.1500 -0.3272 -0.0348  0.6723
-0.0038  0.1928 -0.0982 -0.0151 -0.3948  0.3002 -0.3408
-0.0146  0.4379 -0.1930 -0.0155 -0.3495  0.2122 -0.1522
-0.0241  0.6151 -0.2529 -0.0102 -0.1498 -0.0001  0.2491
-0.0820  0.5299 -0.0793  0.0246  0.6020 -0.3622  0.0380

```

Columns 8 through 9

```

0.0144  0.0637
-0.0493 -0.2428
-0.0088 -0.0241
0.0195  0.0842
0.0583  0.2624
-0.4545  0.6198
0.7615 -0.0180
-0.4496 -0.5199
0.0696  0.4535

```

Note that U and V have the opposite sign from the paper. Since we're multiplying negative by negative, the result is positive. Note that when we multiply the three matrices, the answer is

the original matrix.

```
>> u*s*v'
```

ans =

Columns 1 through 7

1.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000
1.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
1.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	1.0000	1.0000	0.0000	1.0000	0.0000	0.0000
0.0000	1.0000	1.0000	2.0000	0.0000	0.0000	0.0000
0.0000	1.0000	0.0000	0.0000	1.0000	0.0000	0.0000
0.0000	1.0000	0.0000	0.0000	1.0000	0.0000	0.0000
0.0000	1.0000	0.0000	0.0000	1.0000	0.0000	0.0000
0.0000	0.0000	1.0000	1.0000	0.0000	0.0000	0.0000
0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	1.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Columns 8 through 9

0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	1.0000
1.0000	0.0000
1.0000	1.0000
1.0000	1.0000

```
>> u*u'
```

ans =

Columns 1 through 7

0.8871	0.1290	-0.0161	-0.2258	0.0161	0.1129	0.1129
0.1290	0.7097	0.1613	0.2581	-0.1613	-0.1290	-0.1290
-0.0161	0.1613	0.8548	-0.0323	0.1452	0.0161	0.0161
-0.2258	0.2581	-0.0323	0.5484	0.0323	0.2258	0.2258
0.0161	-0.1613	0.1452	0.0323	0.8548	-0.0161	-0.0161
0.1129	-0.1290	0.0161	0.2258	-0.0161	0.3871	0.3871
0.1129	-0.1290	0.0161	0.2258	-0.0161	0.3871	0.3871
0.0806	0.1935	-0.2742	0.1613	0.2742	-0.0806	-0.0806
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Columns 8 through 12

0.0806	0.0000	0.0000	0.0000	0.0000
0.1935	0.0000	0.0000	0.0000	0.0000
-0.2742	0.0000	0.0000	0.0000	0.0000
0.1613	0.0000	0.0000	0.0000	0.0000
0.2742	0.0000	0.0000	0.0000	0.0000
-0.0806	0.0000	0.0000	0.0000	0.0000
-0.0806	0.0000	0.0000	0.0000	0.0000
0.3710	0.0000	0.0000	0.0000	0.0000
0.0000	1.0000	0.0000	0.0000	0.0000
0.0000	0.0000	1.0000	0.0000	0.0000
0.0000	0.0000	0.0000	1.0000	0.0000
0.0000	0.0000	0.0000	0.0000	1.0000

Yipes! that product should have been the identity matrix!

```
>> v*v'
```

ans =

Columns 1 through 7

```

1.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 1.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 1.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 1.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 1.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000 1.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 1.0000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000

```

Columns 8 through 9

```

0.0000 0.0000
0.0000 0.0000
0.0000 0.0000
0.0000 0.0000
0.0000 0.0000
0.0000 0.0000
0.0000 0.0000
0.0000 0.0000
1.0000 0.0000
0.0000 1.0000

```

That one turned out right.

```
>> s
```

```
s =
```

Columns 1 through 7

```

3.3409    0    0    0    0    0    0
 0  2.5417    0    0    0    0    0
 0    0  2.3539    0    0    0    0
 0    0    0  1.6445    0    0    0
 0    0    0    0  1.5048    0    0
 0    0    0    0    0  1.3064    0
 0    0    0    0    0    0  0.8459
 0    0    0    0    0    0    0

```

```
0 0 0 0 0 0 0
```

Columns 8 through 9

```

0 0
0 0
0 0
0 0
0 0
0 0
0 0
0.5601 0
0 0.3637

```

```
>> s(1:2,1:2)
```

```
ans =
```

```

3.3409    0
 0  2.5417

```

```
>> u(:,1:2)
```

```
ans =
```

```

-0.2214 -0.1132
-0.1976 -0.0721
-0.2405  0.0432
-0.4036  0.0571
-0.6445 -0.1673
-0.2650  0.1072
-0.2650  0.1072
-0.3008 -0.1413
-0.2059  0.2736
-0.0127  0.4902
-0.0361  0.6228
-0.0318  0.4505

```

```
>> v(:,1:2)'
```

```
ans =
Columns 1 through 7
-0.1974 -0.6060 -0.4629 -0.5421 -0.2795 -0.0038 -0.0146
-0.0559 0.1656 -0.1273 -0.2318 0.1068 0.1928 0.4379
```

```
Columns 8 through 9
-0.0241 -0.0820
0.6151 0.5299
```

```
>> u(:,1:2)*s(1:2,1:2)*v(:,1:2)'
```

```
ans =
Columns 1 through 7
0.1621 0.4005 0.3790 0.4676 0.1760 -0.0527 -0.1151
0.1406 0.3698 0.3290 0.4004 0.1650 -0.0328 -0.0706
0.1524 0.5050 0.3579 0.4101 0.2362 0.0242 0.0598
0.2580 0.8411 0.6057 0.6974 0.3923 0.0331 0.0832
0.4488 1.2344 1.0509 1.2658 0.5563 -0.0738 -0.1547
0.1596 0.5817 0.3752 0.4169 0.2765 0.0559 0.1322
0.1596 0.5817 0.3752 0.4169 0.2765 0.0559 0.1322
0.2185 0.5496 0.5110 0.6281 0.2425 -0.0654 -0.1425
0.0969 0.5321 0.2299 0.2118 0.2665 0.1368 0.3146
-0.0613 0.2321 -0.1389 -0.2656 0.1449 0.2404 0.5461
-0.0647 0.3353 -0.1456 -0.3014 0.2028 0.3057 0.6949
-0.0431 0.2539 -0.0967 -0.2079 0.1519 0.2212 0.5029
```

```
Columns 8 through 9
-0.1591 -0.0918
-0.0968 -0.0430
0.0869 0.1240
0.1218 0.1874
-0.2096 -0.0489
```

```
0.1889 0.2169
0.1889 0.2169
-0.1966 -0.1079
0.4444 0.4250
0.7674 0.6637
0.9766 0.8487
0.7069 0.6155
```

```
>> save svdeg
>> clear
>> quit
```

```
432432 flops.
```

```
retriever:32>
```

```
retriever:33> matlab
```

< M A T L A B (R) >
(c) Copyright 1984-97 The MathWorks, Inc.
All Rights Reserved
Version 5.1.0.421
May 25 1997

To get started, type one of these commands: helpwin, helpdesk, or demo.
For information on all of the MathWorks products, type tour.

```
>> load svdeg
```

This was the name I saved this example under. See the web page if you want to work with it yourself.

```
>> x
```

```
x =
```

```
1 0 0 1 0 0 0 0 0
```

```

1  0  1  0  0  0  0  0  0
1  1  0  0  0  0  0  0  0
0  1  1  0  1  0  0  0  0
0  1  1  2  0  0  0  0  0
0  1  0  0  1  0  0  0  0
0  1  0  0  1  0  0  0  0
0  0  1  1  0  0  0  0  0
0  1  0  0  0  0  0  0  1
0  0  0  0  0  1  1  1  0
0  0  0  0  0  0  1  1  1
0  0  0  0  0  0  0  1  1

```

The old values of the matrices were stored, so clear those.

```

>> clear u
>> clear s
>> clear v
>> [u,s,v]=svd(x)

```

This is the "pure" form of the SVD, which is fast enough on small matrices like this one.

In general, SVDS is much faster, but less accurate.

u =

Columns 1 through 7

```

0.2214 -0.1132  0.2890 -0.4148 -0.1063 -0.3410  0.5227
0.1976 -0.0721  0.1350 -0.5522  0.2818  0.4959 -0.0704
0.2405  0.0432 -0.1644 -0.5950 -0.1068 -0.2550 -0.3022
0.4036  0.0571 -0.3378  0.0991  0.3317  0.3848  0.0029
0.6445 -0.1673  0.3611  0.3335 -0.1590 -0.2065 -0.1658
0.2650  0.1072 -0.4260  0.0738  0.0803 -0.1697  0.2829
0.2650  0.1072 -0.4260  0.0738  0.0803 -0.1697  0.2829
0.3008 -0.1413  0.3303  0.1881  0.1148  0.2722  0.0330
0.2059  0.2736 -0.1776 -0.0324 -0.5372  0.0809 -0.4669
0.0127  0.4902  0.2311  0.0248  0.5942 -0.3921 -0.2883
0.0361  0.6228  0.2231  0.0007 -0.0683  0.1149  0.1596
0.0318  0.4505  0.1411 -0.0087 -0.3005  0.2773  0.3395

```

Columns 8 through 12

```

-0.0605 -0.4067 -0.0759  0.3227 -0.0551
-0.0099 -0.1089 -0.0339 -0.4564 -0.2844
 0.0623  0.4924  0.1098  0.1337  0.3395
-0.0004  0.0123 -0.1517  0.6453 -0.1102
 0.0343  0.2707 -0.1098 -0.1337 -0.3395
-0.0161 -0.0539 -0.5750 -0.4324  0.3087
-0.0161 -0.0539  0.7267 -0.2129 -0.1986
-0.0190 -0.1653  0.2954 -0.0552  0.7340
-0.0363 -0.5794  0.0000  0.0000  0.0000
 0.2546 -0.2254  0.0000  0.0000  0.0000
-0.6811  0.2320  0.0000  0.0000  0.0000
 0.6784  0.1825  0.0000  0.0000  0.0000

```

s =

Columns 1 through 7

```

3.3409      0      0      0      0      0      0
 0  2.5417      0      0      0      0      0
 0      0  2.3539      0      0      0      0
 0      0      0  1.6445      0      0      0
 0      0      0      0  1.5048      0      0
 0      0      0      0      0  1.3064      0
 0      0      0      0      0      0  0.8459
 0      0      0      0      0      0      0
 0      0      0      0      0      0      0
 0      0      0      0      0      0      0
 0      0      0      0      0      0      0
 0      0      0      0      0      0      0

```

Columns 8 through 9

```

0      0
0      0
0      0

```

```

0      0
0      0
0      0
0      0
0.5601 0
0 0.3637
0      0
0      0
0      0

```

v =

Columns 1 through 7

```

0.1974 -0.0559 0.1103 -0.9498 0.0457 -0.0766 0.1773
0.6060 0.1656 -0.4973 -0.0286 -0.2063 -0.2565 -0.4330
0.4629 -0.1273 0.2076 0.0416 0.3783 0.7244 -0.2369
0.5421 -0.2318 0.5699 0.2677 -0.2056 -0.3689 0.2648
0.2795 0.1068 -0.5054 0.1500 0.3272 0.0348 0.6723
0.0038 0.1928 0.0982 0.0151 0.3948 -0.3002 -0.3408
0.0146 0.4379 0.1930 0.0155 0.3495 -0.2122 -0.1522
0.0241 0.6151 0.2529 0.0102 0.1498 0.0001 0.2491
0.0820 0.5299 0.0793 -0.0246 -0.6020 0.3622 0.0380

```

Columns 8 through 9

```

-0.0144 -0.0637
0.0493 0.2428
0.0088 0.0241
-0.0195 -0.0842
-0.0583 -0.2624
0.4545 -0.6198
-0.7615 0.0180
0.4496 0.5199
-0.0696 -0.4535

```

>> u*u'

ans =

Columns 1 through 7

```

1.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 1.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 1.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 1.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 1.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000 1.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 1.0000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000

```

Columns 8 through 12

```

0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000
1.0000 0.0000 0.0000 0.0000 0.0000
0.0000 1.0000 0.0000 0.0000 0.0000
0.0000 0.0000 1.0000 0.0000 0.0000
0.0000 0.0000 0.0000 1.0000 0.0000
0.0000 0.0000 0.0000 0.0000 1.0000

```

But note that the new u really is orthonormal! That alone makes me suspicious of the SVDS procedure.

>> v*v'

ans =

Columns 1 through 7

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Columns 8 through 9

0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
0.0000	0.0000
1.0000	0.0000
0.0000	1.0000

>>

>> u(:,1:2)*s(1:2,1:2)*v(:,1:2)'

ans =

Columns 1 through 7

0.1621	0.4005	0.3790	0.4676	0.1760	-0.0527	-0.1151
0.1406	0.3698	0.3290	0.4004	0.1650	-0.0328	-0.0706
0.1524	0.5050	0.3579	0.4101	0.2362	0.0242	0.0598
0.2580	0.8411	0.6057	0.6974	0.3923	0.0331	0.0832
0.4488	1.2344	1.0509	1.2658	0.5563	-0.0738	-0.1547
0.1596	0.5817	0.3752	0.4169	0.2765	0.0559	0.1322
0.1596	0.5817	0.3752	0.4169	0.2765	0.0559	0.1322

0.2185	0.5496	0.5110	0.6281	0.2425	-0.0654	-0.1425
0.0969	0.5321	0.2299	0.2118	0.2665	0.1368	0.3146
-0.0613	0.2321	-0.1389	-0.2656	0.1449	0.2404	0.5461
-0.0647	0.3353	-0.1456	-0.3014	0.2028	0.3057	0.6949
-0.0431	0.2539	-0.0967	-0.2079	0.1519	0.2212	0.5029

Columns 8 through 9

-0.1591	-0.0918
-0.0968	-0.0430
0.0869	0.1240
0.1218	0.1874
-0.2096	-0.0489
0.1889	0.2169
0.1889	0.2169
-0.1966	-0.1079
0.4444	0.4250
0.7674	0.6637
0.9766	0.8487
0.7069	0.6155

>> save svdeg

>> quit

24955 flops.

retriever:34>