HOMEWORK 4
COMPUTER SIMULATION OF QUANTUM MEASUREMENT

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Contents

1. Homework Instructions

First construct in Mathematica a random function QMeasure that simulates quantum measurement. The input of QMeasure is an ordered pair $(\Omega, |\psi\rangle)$ [where Ω is an observable of the quantum system given as a matrix in the standard basis and $|\psi\rangle$ is the state of the quantum system given as a column vector written in th standard basis], and whose output with probability $p_j = \langle\psi\,|P_j|\,\psi\rangle$ is the pair $(\lambda_j, |\psi_j\rangle)$ [where λ_j is j-th eigenvalue of Ω, and where $|\psi_j\rangle$ is state of the quantum system resulting from the measurement].

Then for each $N = 100, 500, 1000$, run QMeasure$[\Omega, |\psi\rangle]$ N times to produce a histogram a $\{H_N(1), H_N(2), \ldots, H_N(\nu(\Omega))\}$, where

$$H_N(j) = \frac{\#\lambda_j}{N} = \frac{\#j}{N}\ ,$$

and $\nu(\Omega)$ denotes the number of distinct eigenvalues of the observable Ω. For each N, plot $H_N(j)$ and p_j together, and evaluate how closely $H_N(j)$ approximates p_j.

Do this for each of the inputs given below:

Input1:

$$|\Psi\rangle = (|00\rangle + i\,|01\rangle - |11\rangle)/\sqrt{3} \quad \text{and} \quad \Omega = \begin{pmatrix} 0 & 0 & 1 & -i \\ 0 & 0 & i & -1 \\ 1 & -i & 0 & 0 \\ i & -1 & 0 & 0 \end{pmatrix}$$

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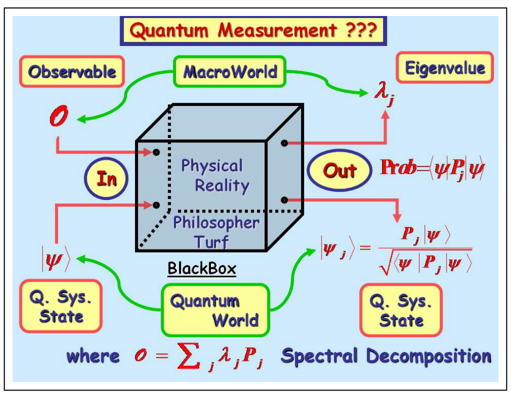
Input2:

$$|\Psi\rangle = (|00\rangle + |01\rangle + |10\rangle + |11\rangle)/2 \quad \text{and} \quad \Omega = \begin{pmatrix} 2 & 0 & 0 & i \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ -i & 0 & 0 & 2 \end{pmatrix}$$

Input3:

$$|\Psi\rangle = (|00\rangle + |01\rangle + |10\rangle + |11\rangle)/2 \quad \text{and} \quad \Omega = \frac{1}{2}\begin{pmatrix} 5 & 0 & 0 & 3i \\ 0 & 5 & i & 0 \\ 0 & -i & 5 & 0 \\ -3i & 0 & 0 & 5 \end{pmatrix}$$

2. Helpful suggestions



Be sure that the Mathematical function QMEASURE$[\Omega, |\psi\rangle]$ normalizes $|\psi\rangle$ to unit length, otherwise your probabilities will be incorrect. *[Please also keep in mind that the orthonormal basis of an eigenspace of dimension greater one is not unique.]*

The Mathematica function QMEASURE$[\Omega, |\psi\rangle]$ will need to call the following Mathematical functions (which you will create):

- SPECTRALDECOMP
  Input: Ω
  Output: $\left\{ \{\lambda_1, \lambda_2, \dots, \lambda_{\nu(\Omega)}\}, \{P_1, P_2, \dots, P_{\nu(\Omega)}\} \right\}$
  % Please construct so that eigenvalues are in ascending numerical order, i.e.,
  $$j_1 < j_2 \Longrightarrow \lambda_{j_1} < \lambda_{j_2}$$

- ComputeProb
  Input: $\{\{|\psi\rangle\},\{P_1, P_2, \ldots, P_{\nu(\Omega)}\}\}$
  Output: $\{p_1, p_2, \ldots, p_{\nu(\Omega)}\}$
  % where $p_j = \langle\psi\,|P_j|\,\psi\rangle$

- ProbRandGen
  Input: $\{p_1, p_2, \ldots, p_{\nu(\Omega)}\}$
  Output: j with probabilty p_j
  % $j = 1, 2, \ldots, \nu(\Omega)$

Here is a suggestion for ProbRandGen written in pseudo code:

$r = RandomReal[]$
$s = 0$
$Loop \quad j = 1 \ldots \nu(\Omega)$
$s = s + p_j$
$If \quad r \leq s \quad Then$
$Output[j]$ and $exit$
$End\ If$
$End\ Loop$

References

[1] Lomonaco, Samuel J., Jr., **"A Rosetta Stone for Quantum Mechanics with an Introduction to Quantum Computation,"** in PSAPM, vol. 58, AMS, Providence, RI, (2002), pp3 - 65.
AMS PDF reprint: https://www.csee.umbc.edu/~lomonaco/pubs/psapm564.pdf
PowerPoint Talk:
http://www.csee.umbc.edu/~lomonaco/slides/GWU-2009-RosettaStone.pdf

[2] Rieffel, Eleanor, and Wolfgang Polak, **"Quantum Computing: A Gentle Introduction,"** MIT Press, (2011).

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