AN EXAMPLE OF A GARNER'S ALGORITHM CALCULATION

DR. LOMONACO

Question: Use Garner's algorithm to find the unique integer $0 \leq x < 5 \cdot 7 \cdot 11$ that satisfies the following three modular equations:

$$\begin{cases} x = 4 \bmod 5 \\ x = 1 \bmod 7 \\ x = 2 \bmod 11 \end{cases}$$

The mixed radix representation of the unigue integer x is of the form

$$x = \nu_0 + \nu_1 \cdot 5 + \nu_2 \cdot 5 \cdot 7$$

Hence, the solution is found by determing the integers ν_0, ν_1, and ν_2 as follows:

$x = 4 \bmod 5 \Longrightarrow \nu_0 + \nu_1 \cdot 5 + \nu_2 \cdot 5 \cdot 7 = 4 \bmod 5 \Longrightarrow \boxed{\nu_0 = 4 \bmod 5}$
$\therefore \boxed{x = 4 + \nu_1 \cdot 5 + \nu_2 \cdot 5 \cdot 7}$

$x = 1 \bmod 7 \Longrightarrow 4 + \nu_1 \cdot 5 + \nu_2 \cdot 5 \cdot 7 = 1 \bmod 7 \Longrightarrow 4 + 5\nu_1 = 1 \bmod 7$
$\Longrightarrow 5\nu_1 = -3 = 4 \bmod 7$
But $5^{-1} \bmod 7 = 3$. Hence, $\boxed{\nu_1 = 12 = 5 \bmod 7}$
$\therefore \boxed{x = 4 + 5 \cdot 5 + \nu_2 \cdot 5 \cdot 7}$

$x = 2 \bmod 11 \Longrightarrow 4 + 5 \cdot 5 + \nu_2 \cdot 5 \cdot 7 = 2 \bmod 11 \Longrightarrow 29 + 35\nu_2 = 2 \bmod 11$
$\Longrightarrow 7 + 2\nu_2 = 2 \bmod 11 \Longrightarrow 2\nu_2 = -5 = 6 \bmod 11$
But $2^{-1} \bmod 11 = 6$. Hence, $\boxed{\nu_2 = 36 = 3 \bmod 11}$
$\boxed{x = 4 + 5 \cdot 5 + 3 \cdot 5 \cdot 7 = 29 + 105 = 134}$

$\boxed{\text{Hence, the answer is } x = 134 \mod 5 \cdot 7 \cdot 11}$