CMSC 655 Project
The Skydiving Computer Hacker's Dilemma

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X, who has just completed CMSC 655, decided that one good way to celebrate the occasion would be to try out skydiving. A friend Y graciously loans X the latest in skydiving equipment, and quickly instructs X in skydiving. X then promptly hops onto a plane, and enthusiastically jumps out. *However*, just after jumping out of the plane, X realizes that he/she forgot to ask Y at what time t_r the ripcord should be pulled.

Fortunately, X realizes that this is just the time when the creative problem solving techniques learned in CMSC 655 would come in handy. Being a dedicated Computer Science student, X always carries with him/her a 500 Megahertz pentium III based notebook computer fully loaded with the latest versions of Maple and MatLab. X also whips out an speedometer and an altimeter (fortunately left in X's skydiving outfit by Y), and finds that at that time (which X cleverly denotes as time $t_0 = 0$), X is $1,000$ meters up and falling at the rate of $v(0) = -7$ meters per second[1].

X recalls from CMSC 655 that the equations for X's velocity $v(t)$ at time t and X's distance $s(t)$ from the ground at time t are:

$$\frac{d}{dt}\begin{pmatrix} v \\ s \end{pmatrix} = \begin{pmatrix} g + K(t)\left|v(t)\right|^{2.1} \\ v(t) \end{pmatrix} \qquad \text{(Equation 1)}$$

where

$$g = -9.8 \quad meters/s^2$$

is the acceleration of gravity and where the coefficient of friction $K(t)$ is given by:

$$K(t) = \begin{cases} 0.01 & \text{in free fall, i.e. for } t \leq t_r \\ 0.21 & \text{with open parachute, i.e., for } t > t_r \end{cases}$$

For the sake of thrills, X decides he wants to pull the ripcord at the last possible moment in order to land on the ground at -7 meters per second.

[1] The positive s direction is upward. So the velocity $v = ds/dt$ is negative because X is falling, i.e., $ds \approx \Delta s < 0$ and $dt \approx \Delta t > 0$. If the velocity were positive, X would be rising.

X recalls that Euler's method can be used to solve the above equation numerically. Specifically, the Euler iteration is given by:

$$\begin{pmatrix} v_{i+1} \\ \\ s_{i+1} \end{pmatrix} = \begin{pmatrix} v_i \\ \\ s_i \end{pmatrix} + h \begin{pmatrix} g + K_i \left| v_i \right|^{2.1} \\ \\ v_i \end{pmatrix} \qquad \text{(Equation 2)}$$

For various choices of h, for $h = 2^{-j}$, $j = 0, 1, 2, 3, \ldots$, compute t_r.

Hint 1: For free fall, numerically compute the function

$$t \longmapsto (v, s)$$

forward in time from $(v_0, s_0) = (-7, 1000)$ to determine a v versus s curve. On the other hand, for the controlled fall with a parachute, numerically compute the function

$$t \longmapsto (v, s)$$

backward[2] in time from $(v_{TERMINAL}, s_{TERMINAL}) = (-7, 0)$ to determine a v versus s curve. Then graph both curves to determine the (v, s)-point at which they intersect [namely, (v_r, s_r)], and finally the time t_r at which they intersect.

Hint 2: Try to see if your answers are in the "ballpark" by analytically solving the problem with the exponent 2.1 replaced with 2.

Next consider the differential equation

$$\frac{dv}{ds} = \frac{g + K(s) \left| v(s) \right|^{2.1}}{v} \qquad \text{(Equation 3)}$$

which gives v as a function of s, where the coefficient $K(s)$ is given by

$$K(s) = \begin{cases} 0.01 & \text{in free fall, i.e. for } s \geq s_r \\ \\ 0.21 & \text{with open parachute, i.e., for } s < s_r \end{cases}$$

and where s_r is the altimeter reading when the chute first opens. The corresponding Euler iteration is:

$$v_{j+1} = v_j + \overline{h} \left(\frac{g + K_j \left| v_j \right|^{2.1}}{v_j} \right) \qquad \text{(Equation 4)}$$

[2] For backward in time, h would be negative. Moreover, given v_i and s_i, one would use Euler's method to find v_{i-1} and s_{i-1}.

Determine the altimeter reading s_r when the chute first opens. For various choices of $\overline{h}$, for $\overline{h} = 2^{-j}$, $j = 0, 1, 2, 3, \ldots$, compute s_r. Compare the accuracy of your results computed with equations 1 and 2 with results computed from equations 3 and 4.

Can you think of a better way to numerically compute t_r and s_r?

After the above calculations, if you were X, would you pull the rip cord on a cue from your watch (i.e., using t_r), or would pull the ripcord on a cue from your altimeter (i.e., using s_r)? (We assume that both the watch and altimeter give error free readings.)

Solve this problem using MatLab. (You may, if you wish, use Maple to verify and analyze your numerical MatLab results.) You may use various plots to demonstrate what is happening both numerically and physically. How accurate are your answers? How does your accuracy change with h and $\overline{h}$?

Be sure to begin your project report with an executive summary and with a table of contents. You will be graded on your clarity of exposition, on the correctness of your analytical thinking, on your demonstrated understanding and insight into the project problem, on your originality and creativity, and on whether you have clearly demonstrated that you are ready for skydiving.