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How to Build a Device that Cannot Be Built

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PowerPoint slides can be found at:
www.csee.umbc.edu/~lomonaco/Lectures.html

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This talk is based on the following paper

Lomonaco, Samuel J., *How to build a device that cannot be built*, Journal of Quantum Information Processing, 15, 3, (2016), pp 1043-1056.

http://www.csee.umbc.edu/~lomonaco/pubs/Unbuildable-QIP.pdf

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Which in turn is based on the following papers:

Mermin, N David, *Quantum Mysteries Revisited*, Am. J. Phys. 58, 750 (1990), pp 731-734.

Greenberger, D.M., M. Horne, & A. Zeilenger, *Going beyond Bell's theorem*, in *"Bell's theorem, Quantum Theory, & Conceptions of the Universe,"* ed. by M. Kafatos, (Kluwer Academic, Dordrecht, 1989),

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The Meaning of Quantum Mechanics ???

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Interpretation of Quantum Physics ???

Meaning of Q.M. ?

Group of Angry Physicists

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The Dilemma of Quantum Physics ???

The Quantum Physics Conundrum:

- There is almost universal agreement in regard to the laws of quantum mechanics.
- But there is almost total disagreement as to their interpretation.

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Interpretations of Quantum Mechanics

- Copenhagen Interpretation
- Many Worlds Interpretation
- Consistent Histories Interpretation
- Ensemble/Statistical Interpretation
- Etc.
- Irish Pub Interpretation

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Dowling's Irish Pub Interpretation

- After 0 Beers: Clear as mud
- After 1 Beer: Somewhat clear
- After 2 Beers: Very clear
- After 3 Beers: Obvious
- After 4 Beers: Puzzling
- After 5 Beers: Nonsense

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What is Q.M. Trying to tell Us ???

- Heisenberg's Uncertainty Principle
  Simultaneous measurement of incompatible observables to unlimited precision not possible
- Bell Inequalities – Violation of the Principle of Reality, or the Principle of Locality
- Kochen-Specker Theorem - Ditto
- Conway-Kochen Free Will Theorem – The past does not determine the future.

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Objective of Talk

Objective: To investigate the Greenberger-Horne-Zeillenger (GHZ) Paradox using the tools of Quantum Information Science; and to push this investigation to its limits.

One Result: The second elementary symmetric Boolean function σ_2 can be interpreted as a quantification of the nonlocality and/or indeterminism involved in the GHZ paradox.

We will also illustrate the subtlety of the distributed control of distributed quantum systems.

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As Always …

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A Connundrum

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The Story:
A Tale of 3 Regions

Enter Stage Right: Supervisor

Build this physical System.

Or Else !!!   ASAP !!!

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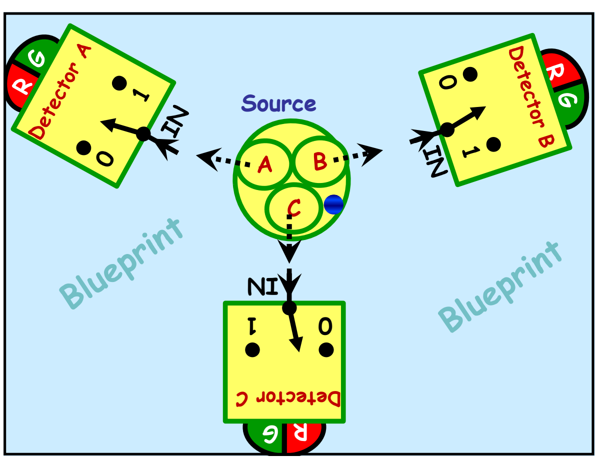

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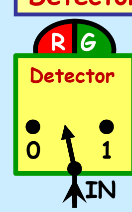
Detector



- Upon encountering a particle, the detector flashes either red R=1 or green G=0
- Each detector has a switch with two settings 0 and 1, which can be randomly set at anytime before the particle arrives.

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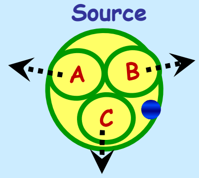
Source S



- A source containg 3 particles, and a little blue button, which when pressed, ejects the particles A, B, C toward their respective detectors A, B, & C

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The supervisor is only interested in the switch settings for which an odd number of the three switches is set to 1 ,i.e.,

A B C	A B C	A B C	A B C
0 0 1	0 1 0	1 0 0	1 1 1

No other switch settings are important. He/she doesn't care about remaining 4 settings.

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Specifications:

Spec. 1:
For switch settings 001, 010, 100 (after all 3 particles received), ONLY an odd number of the detectors flash RED R=1

Spec. 2:
For switch setting 111 (after all particles received), ONLY an even number of detectors must flash RED R=1

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Constraints

Constraint 1:
Detectors cannot communicate with one another. They are separated by a spacelike distance, and hence physically independent.

Constraint 2:
Upon leaving source, the particles can no longer communicate with one another.

Constraint 3:
Each particle only communicates with a detector when it encounters the detector.

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Supervisor leaves with an ominous command:

Build this system ASAP !!!

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A Design Engineer's Observations:

Enter stage left: Design Engineer

- Since:
  1) Detectors cannot communicate
  2) After ejection, particles cannot communicate
  3) Particles only communicate with detector upon impact
- Ergo, each particle must carry instructions for its respective detector.

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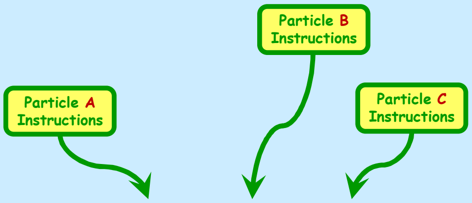
A Design Engineer's Observations:



Switch \ Ball	A	B	C
1	Color1A	Color1B	Color1C
2	Color2A	Color 2B	Color2C

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Each particle must carry its on instructions for its detector.

Thus, particle **A** must carry a local instruction of $f_A(s_A)$ of the form:

$$f_A(s_A) = \begin{cases} c_{A0} & \textit{if switch setting } s_A = 0 \\ c_{A1} & \textit{if switch setting } s_A = 1 \end{cases}$$

where $c_{A0} = R(=1) \textit{ or } G(=0)$ and $c_{A1} = R(=1) \textit{ or } G(=0)$

In like manner, the remaining two particles must carry instructions $f_B(s_B)$ and $f_C(s_C)$

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Thus, for j = **A**, **B**, **C**, each local instruction is simply a Boolean function of the form:

$$f_j : \{0,1\} \to \{0,1\}$$

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Specifications:

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$$\begin{cases} f_A(0) + f_B(0) + f_C(1) = 1(\bmod 2) \\ f_A(0) + f_B(1) + f_C(0) = 1(\bmod 2) \\ f_A(1) + f_B(0) + f_C(0) = 1(\bmod 2) \end{cases} \text{Spec 1}$$

$$f_A(1) + f_B(1) + f_C(1) = 0(\bmod 2) \quad \text{Spec 2}$$

The above system of linear equations is obviously inconsistent !!!

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Ergo,

It can't be built !!!

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Oh, but it can be built !!!

Enter Stage Right:
A Quantum Computer Scientist

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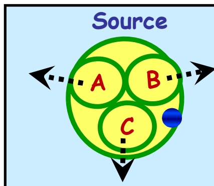
Source S



Let the 3 particles in the source be photons in the entangled state:

$$|\psi\rangle = \frac{1}{2}\left(|000\rangle - |011\rangle - |101\rangle - |110\rangle\right)$$

where $|0\rangle$ denotes a horizontally polarized state, & $|1\rangle$ denotes a vertically polarized state

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Please note that

$$|\psi\rangle = \frac{1}{2}\left(|000\rangle - |011\rangle - |101\rangle - |110\rangle\right)$$

is **entangled**. It cannot be factored into the tensor product of 3 separate qubit states.

The state of each qubit is **indeterminate**. But the state of all 3 is well defined !!!

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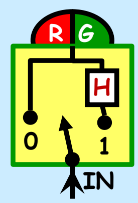
The Detector



We insert a Hadamard transform at switch setting 2 .

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Hence,

$$H|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$H|1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

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Definition. A **Boolean unitary transformation** is a map from $\{0,1\}^k$ into a group of unitary transformations. In like manner, a **Boolean Hermitian operator** is a map from $\{0,1\}^k$ into an algebra of observables.

In other words, Boolean unitary and Boolean Hermitian operators are unitary and Hermitian transformations controlled by classical bits.

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Boolean Unitaries and Boolean Observables

If b (= 0 or 1) and if U is a unitary operator, then U^b will denote the **Boolean unitary**

$$U^b = \begin{cases} I & if \quad b = 0 \\ U & if \quad b = 1 \end{cases}$$

where I is the identity operator.

In like manner, if Ω is an observable, then $b\Omega$ will denote the Boolean observable

$$b\Omega = \begin{cases} 0 & if \quad b = 0 \\ \Omega & if \quad b = 1 \end{cases}$$

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The following wiring diagram describes how the unbuildable device can be built in a quantum computer laboratory:

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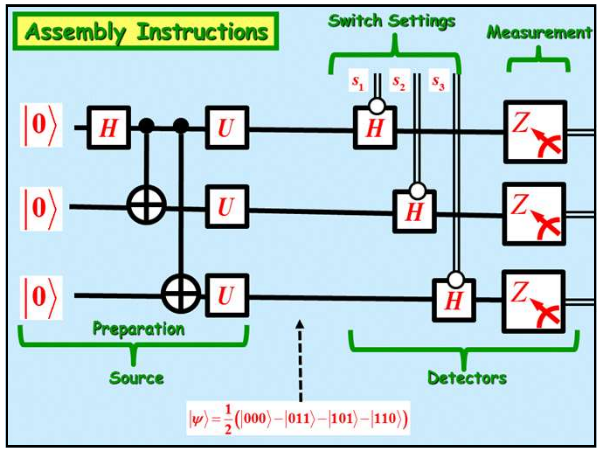

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How to read the wiring diagram

- **Single lines** represent qubits, and **double lines** represent classical bits

-  denotes the **Controlled-NOT** gate, a.k.a., **CNOT** given by the unitary transformation:

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

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How to read the wiring diagram

- 

denotes **measurement** with respect to the standard basis $|0\rangle$, $|1\rangle$, i.e., measurement with respect to the Pauli spin operator **Z** , where

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

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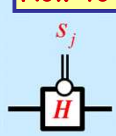
How to read the wiring diagram

- 

denotes the Boolean unitary op.

$$H^{s_j^*} = \begin{cases} I & if \quad s_j^* = 0 \Leftrightarrow s_j = 1 \\ H & if \quad s_j^* = 1 \Leftrightarrow s_j = 0 \end{cases}$$

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How to read the wiring diagram

Where U denotes the gate

$$U = \exp\left[\frac{i\pi}{3}\left(\frac{X+Y+Z}{\sqrt{3}}\right)\right] = \frac{1+i}{2}\begin{pmatrix} 1 & 1 \\ 1 & -i \end{pmatrix}$$

and where **X**, **Y**, **Z** denote the Pauli spin operators:

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

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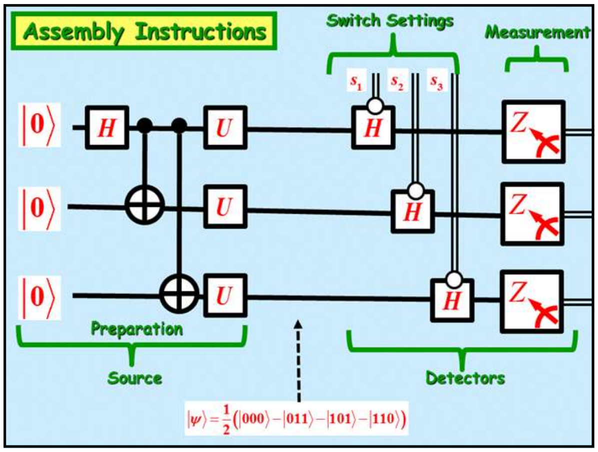

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A Momentary Digression

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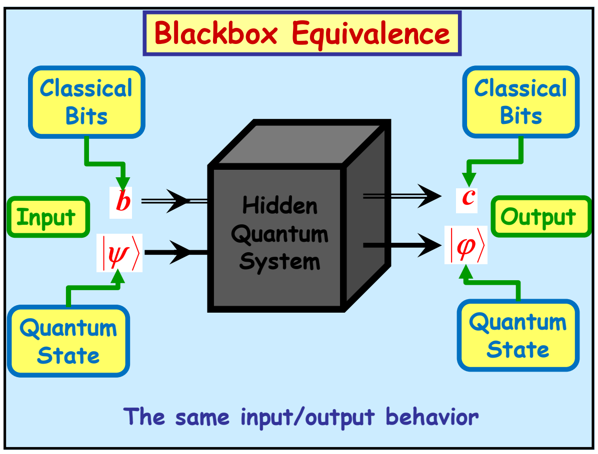

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Simplifying the diagram

We can simplify the diagram by using the measurement gate s_j

which is defined by

$$\Upsilon(s_j) = s_j^* X + s_j Z = \begin{cases} X & if \quad s_j = 0 \\ Z & if \quad s_j = 1 \end{cases}$$

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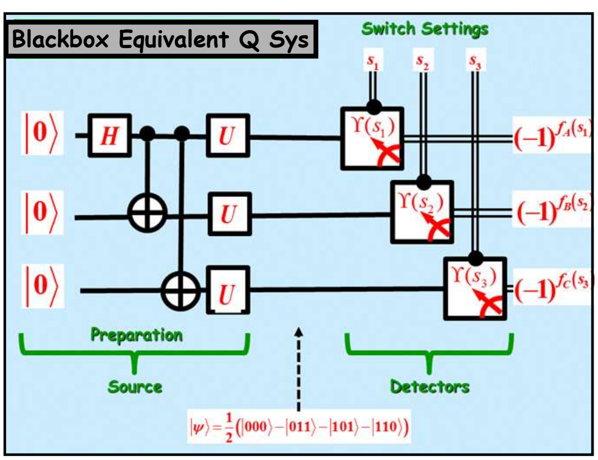

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Back to the original Assembly Instructions

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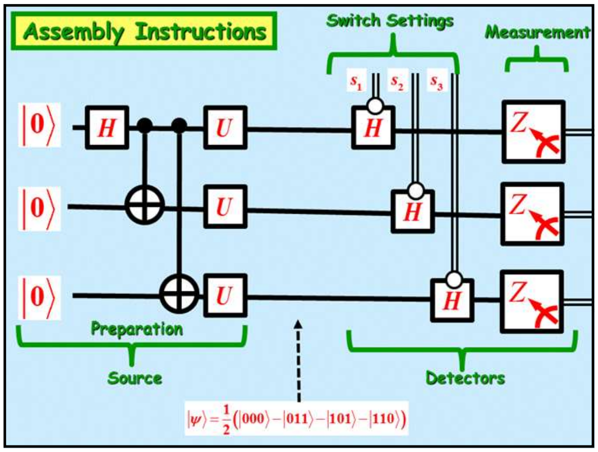

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What do the 3 detectors see for each of the 4 possible legal switch settings ???

Switch Settings $s=(s_1,s_2,s_3)$	State $\left(H^{s_1}\otimes H^{s_2}\otimes H^{s_3}\right)\lvert\psi\rangle$
111	$(1\otimes 1\otimes 1)\lvert\psi\rangle = \frac{1}{2}(\lvert 000\rangle - \lvert 011\rangle - \lvert 101\rangle - \lvert 110\rangle)$
001	$(H\otimes H\otimes 1)\lvert\psi\rangle = \frac{1}{2}(-\lvert 001\rangle + \lvert 010\rangle + \lvert 100\rangle + \lvert 111\rangle)$
010	$(H\otimes 1\otimes H)\lvert\psi\rangle = \frac{1}{2}(\lvert 001\rangle - \lvert 010\rangle + \lvert 100\rangle + \lvert 111\rangle)$
100	$(1\otimes H\otimes H)\lvert\psi\rangle = \frac{1}{2}(\lvert 001\rangle + \lvert 010\rangle - \lvert 100\rangle + \lvert 111\rangle)$

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What do the 3 detectors see for switch settings 111 ?

$$|\psi\rangle = \frac{1}{2}(|000\rangle - |011\rangle - |101\rangle - |110\rangle)$$

p = 1/4

000	011	101	110
GGG	GRR	RGR	RRG

Therefore, <u>always</u> an even number of REDs R

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What do the 3 detectors see for switch settings 001 ?

$$(H\otimes H\otimes 1)|\psi\rangle = \frac{1}{2}(-|001\rangle + |010\rangle + |100\rangle + |111\rangle)$$

p = 1/4

001	010	100	111
GGR	GRG	RGG	RRR

Therefore, <u>always</u> an odd number of REDs R

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Why ???

So where has the proof of impossibility gone awry?

The proof of impossibility is based on the following proposition:

<u>Proposition</u>: There exist no set of Boolean functions

$f_A:\{0,1\}\to\{0,1\},\ f_B:\{0,1\}\to\{0,1\},\ f_B:\{0,1\}\to\{0,1\}$

such that

$$f_A(s_1)+f_B(s_2)+f_C(s_3)=\begin{cases}1(\text{mod}\,2) & \text{if} \quad s=001,010,100\\ 0(\text{mod}\,2) & \text{if} \quad s=111\end{cases}$$

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The logic is flawless !!!

But the crux of the matter is that an argument is **only** as sound as the assumptions upon which it is based.

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More explicitly, the argument fails because at least one of the following tacitly assumed two assumptions fails:

Premise 1. Reality Principle: What is measured is completely determined before it is measured.

Premise 2. Principle of Locality: Spacelike separated regions of spacetime are physically independent.

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The above two premises lead to the following unfounded conclusions:

Unfounded Conclusion 1. Based on Premise 1 (The Reality Principle), the detector lamp instructions f_A, f_B, f_C must already be predetermined well-defined total functions at the time of particle ejection.

Unfounded Conclusion 2. Based on Premise 2 (The Principle of Locality), the detector lamp instructions f_A, f_B, f_C must be local. Hence, f_j is a function only of the j-th switch setting s_j and independent of the two other switch settings.

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Corollary 1. For a switch settings $s = (s_1, s_2, s_3)$ of odd Hamming weight, the detector lamp instructions f_A, f_B, f_C are the random partial functions given by:

$$\begin{cases} f_A(s) = j_1 \\ f_B(s) = j_2 \\ f_C(s) = j_3 \end{cases}$$

with the Boolean algebraic dependence

$$f_A(s) + f_B(s) + f_C(s) = \sigma_2(s_1, s_2, s_3) + 1 \pmod 2$$

where σ_2 denotes the second elementary symmetric function

$$\sigma_2(s_1, s_2, s_3) = s_1 \cdot s_2 + s_2 \cdot s_3 + s_3 \cdot s_1$$

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In other words, the GHZ paradox shows how to create three spacelike separated (hence, physically independent) probability distributions that have the above algebraic dependence.

The Boolean function σ_2 quantifies the nonlocality of the GHZ paradox.

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