How to Build a Device that Cannot Be Built

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L-O-O-P Fund

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PowerPoint slides can be found at:
www.csee.umbc.edu/~lomonaco/Lectures.html

This talk is based on the following paper

Lomonaco, Samuel J., *How to build a device that cannot be built*, Journal of Quantum Information Processing, 15, 3, (2016), pp 1043-1056.

http://link.springer.com/article/10.1007/s11128-015-1206-7

Which in turn is based on the following papers:

Mermin, N David, *Quantum Mysteries Revisited*, Am. J. Phys. 58, 750 (1990), pp 731-734.

Greenberger, D.M., M. Horne, & A. Zeilenger, *Going beyond Bell's theorem*, in *"Bell's theorem, Quantum Theory, & Conceptions of the Universe,"* ed. by M. Kafatos, (Kluwer Academic, Dordrecht, 1989),

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The Meaning of Quantum Mechanics ???

Interpretation of Quantum Physics ???

Meaning of Q.M. ?

Group of Angry Physicists

The Dilemma of Quantum Physics ???

The Quantum Physics Conundrum:

- There is almost universal agreement in regard to the laws of quantum mechanics.
- But there is almost total disagreement as to their interpretation.

Interpretations of Quantum Mechanics

- Copenhagen Interpretation
- Many Worlds Interpretation
- Consistent Histories Interpretation
- Ensemble/Statistical Interpretation
- Etc.
- Irish Pub Interpretation

Dowling's Irish Pub Interpretation

- After 0 Beers: Clear as mud
- After 1 Beer: Somewhat clear
- After 2 Beers: Very clear
- After 3 Beers: Obvious
- After 4 Beers: Puzzling
- After 5 Beers: Nonsense

What is Q.M. Trying to tell Us ???

- Heisenberg's Uncertainty Principle
  Simultaneous measurement of incompatible observables to unlimited precision not possible
- Bell Inequalities – Violation of the Principle of Reality, or the Principle of Locality
- Kochen-Specker Theorem - Ditto
- Conway-Kochen Free Will Theorem – The past does not determine the future.

Objective of Talk

Objective: To investigate the Greenberger-Horne-Zeillenger (GHZ) Paradox using the tools of Quantum Information Science; and to push this investigation to its limits.

One Result: The second elementary symmetric Boolean function σ_2 can be interpreted as a quantification of the nonlocality and/or indeterminism involved in the GHZ paradox.

We will also illustrate the subtlety of the distributed control of distributed quantum systems.

As Always …

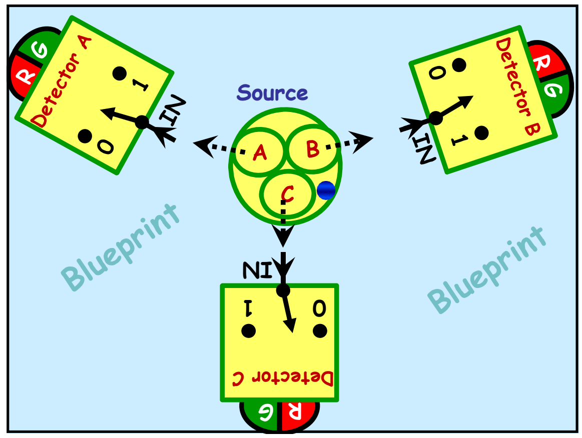
A Connundrum

The Story: A Tale of 3 Regions

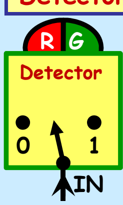
Enter Stage Right: Supervisor

Build this physical System.

Or Else !!! ASAP !!!

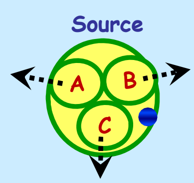


Detector



- Upon encountering a particle, the detector flashes either red R=1 or green G=0
- Each detector has a switch with two settings 0 and 1, which can be randomly set at anytime before the particle arrives.

Source S



- A source containg 3 particles, and a little blue button, which when pressed, ejects the particles A, B, C toward the respective detectors A, B, & C

The supervisor is only interested in the switch settings for which an odd number of the three switches is set to 1 ,i.e.,

A B C	A B C	A B C	A B C
0 0 1	0 1 0	1 0 0	1 1 1

No other switch settings are important. He/she doesn't care about remaining 4 settings.

Specifications:

Spec. 1:
For switch settings 001, 010, 100 (after all 3 particles received), ONLY an odd number of the detectors flash RED R=1

Spec. 2:
For switch setting 111 (after all particles received), ONLY an even number of detectors must flash RED R=1

Constraints

Constraint 1:
Detectors cannot communicate with one another. They are separated by a spacelike distance, and hence physically independent.

Constraint 2:
Upon leaving source, the particles can no longer communicate with one another.

Constraint 3:
Each particle only communicates with a detector when it encounters the detector.

Supervisor leaves with an ominous command:

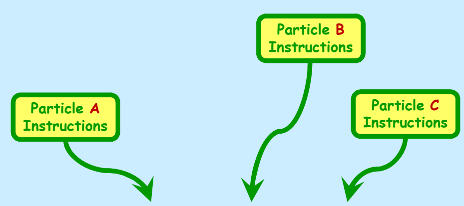
Build this system ASAP !!!

A Design Engineer's Observations:

Enter stage left: Design Engineer

- Since:
  1) Detectors cannot communicate
  2) After ejection, particles cannot communicate
  3) Particles only communicate with detector upon impact
- Ergo, each particle must carry instructions for its respective detector.

A Design Engineer's Observations:



Switch \ Ball	A	B	C
1	Color1A	Color1B	Color1C
2	Color2A	Color 2B	Color2C

Each particle must carry its on instructions for the its detector.

Thus, particle **A** must carry a local instruction of $f_A(s_A)$ of the form:

$$f_A(s_A) = \begin{cases} c_{A0} & \text{if switch setting } s_A = 0 \\ c_{A1} & \text{if switch setting } s_A = 1 \end{cases}$$

where $c_{A0} = R(=1) \text{ or } G(=0)$ and $c_{A1} = R(=1) \text{ or } G(=0)$

In like manner, the remaining two particles must carry instructions $f_B(s_B)$ and $f_C(s_C)$

Thus, for **j** = **A**, **B**, **C**, each local instruction is simply a Boolean function of the form:

$$f_j : \{0,1\} \to \{0,1\}$$

Specifications:

Spec. 1:
For switch settings **001**, **010**, **100** (after all 3 particles received), ONLY an odd number of the detectors flash **RED R=1**

Spec. 2:
For switch setting **111** (after all particles received), ONLY an even number of detectors must flash **RED R=1**

$$\left\{\begin{array}{l} \left.\begin{array}{l} f_A(0) + f_B(0) + f_C(1) = 1 \pmod 2 \\ f_A(0) + f_B(1) + f_C(0) = 1 \pmod 2 \\ f_A(1) + f_B(0) + f_C(0) = 1 \pmod 2 \end{array}\right\} \text{Spec 1} \\ \left. f_A(1) + f_B(1) + f_C(1) = 0 \pmod 2 \right\} \text{Spec 2} \end{array}\right.$$

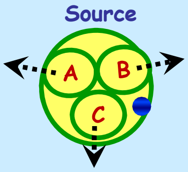
The above system of linear equations is obviously **inconsistent** !!!

Ergo,

It can't be built !!!

Oh, but it can be built !!!

Enter Stage Right:
A Quantum Computer Scientist



Source S

Let the 3 particles in the source be photons in the entangled state:

$$|\psi\rangle = \frac{1}{2}\left(|000\rangle - |011\rangle - |101\rangle - |110\rangle\right)$$

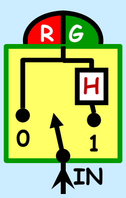
where $|0\rangle$ denotes a horizontally polarized state, & $|1\rangle$ denotes a vertically polarized state

Please note that

$$|\psi\rangle = \frac{1}{2}\left(|000\rangle - |011\rangle - |101\rangle - |110\rangle\right)$$

is entangled. It cannot be factored into the tensor product of 3 separate qubit states.

The state of each qubit is indeterminate. But the state of all 3 is well defined !!!



The Detector

We insert a Hadamard transform at switch setting 2 .

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Hence,

$$\begin{cases} H|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \\ H|1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) \end{cases}$$

Definition. A Boolean unitary transformation is a map from $\{0,1\}^k$ into a group of unitary transformations. In like manner, a Boolean Hermitian operator is a map from $\{0,1\}^k$ into an algebra of observables.

In other words, Boolean unitary and Boolean Hermitian operators are unitary and Hermitian transformations controlled by classical bits.

Boolean Unitaries and Boolean Obsevables

If b (= 0 or 1) and if U is a unitary operator, then U^b will denote the Boolean unitary

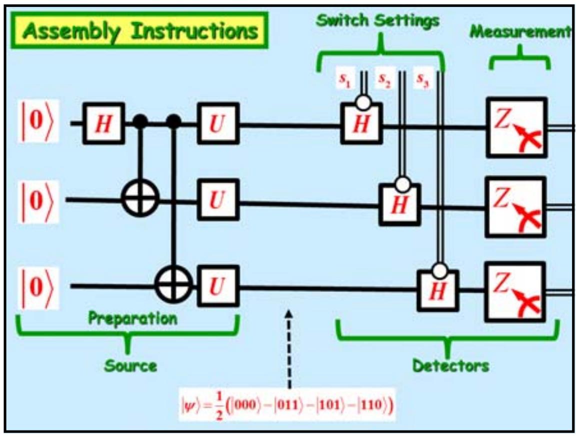
$$U^b = \begin{cases} I & if \quad b = 0 \\ U & if \quad b = 1 \end{cases}$$

where I is the identity operator.

In like manner, if Ω is an observable, then $b\Omega$ will denote the Boolean observable

$$b\Omega = \begin{cases} 0 & if \quad b = 0 \\ \Omega & if \quad b = 1 \end{cases}$$

The following wiring diagram describes how the unbuildable device can be built in a quantum computer laboratory:



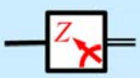
How to read the wiring diagram

- Single lines represent qubits, and double lines represent classical bits

-  denotes the Controlled-NOT gate, a.k.a., CNOT given by the unitary transformation:

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

How to read the wiring diagram

-  denotes measurement with respect to the standard basis $|0\rangle$, $|1\rangle$, i.e., measurement with respect to the Pauli spin operator **Z**.

How to read the wiring diagram

-  denotes the Boolean observable

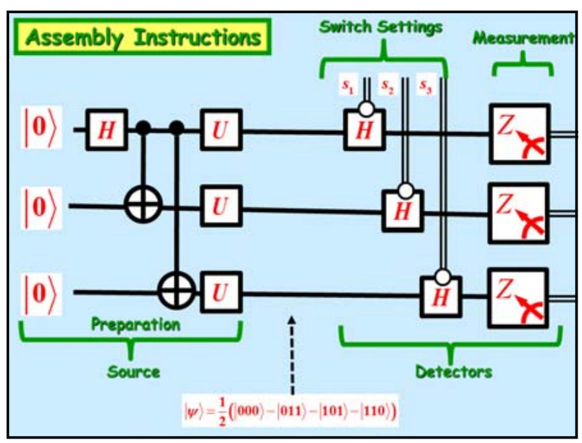
$$H^{s_j^*} = \begin{cases} I & if \quad s_j^* = 0 \Leftrightarrow s_j = 1 \\ H & if \quad s_j^* = 1 \Leftrightarrow s_j = 0 \end{cases}$$

How to read the wiring diagram

Where U denotes the gate

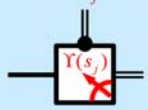
$$U = \exp\left[\frac{i\pi}{3}\left(\frac{X+Y+Z}{\sqrt{3}}\right)\right] = \frac{1+i}{2}\begin{pmatrix} 1 & 1 \\ 1 & -i \end{pmatrix}$$

and where **X**, **Y**, **Z** denote the Pauli spin operators.



Simplifying the diagram

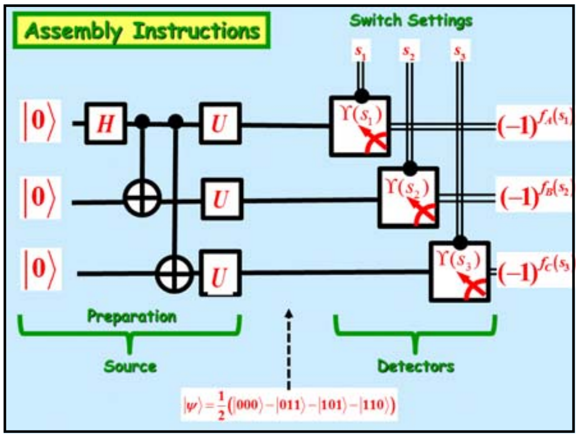
We can simplify the diagram by using the measurement gate s_j



which is defined by

$$\Upsilon(s_j) = s_j^* X + s_j Z = \begin{cases} X & \textit{if} \quad s_j = 0 \\ Z & \textit{if} \quad s_j = 1 \end{cases}$$

Assembly Instructions



What do the 3 detectors see for each of the 4 possible legal switch settings ???

Switch Settings $s=(s_1,s_2,s_3)$	State $\left(H^{s_1^*} \otimes H^{s_2^*} \otimes H^{s_3^*}\right)\|\psi\rangle$
111	$(1\otimes 1\otimes 1)\|\psi\rangle = \frac{1}{2}\left(\|000\rangle - \|011\rangle - \|101\rangle - \|110\rangle\right)$
001	$(H\otimes H\otimes 1)\|\psi\rangle = \frac{1}{2}\left(-\|001\rangle + \|010\rangle + \|100\rangle + \|111\rangle\right)$
010	$(H\otimes 1\otimes H)\|\psi\rangle = \frac{1}{2}\left(\|001\rangle - \|010\rangle + \|100\rangle + \|111\rangle\right)$
100	$(1\otimes H\otimes H)\|\psi\rangle = \frac{1}{2}\left(\|001\rangle + \|010\rangle - \|100\rangle + \|111\rangle\right)$

What do the 3 detectors see for switch settings 111 ?

$$|\psi\rangle = \frac{1}{2}\left(|000\rangle - |011\rangle - |101\rangle - |110\rangle\right)$$

p = 1/4

000	011	101	110
GGG	GRR	RGR	RRG

Therefore, <u>always</u> an even number of REDs R

What do the 3 detectors see for switch settings 001 ?

$$|\psi\rangle = \frac{1}{2}\left(H\otimes H\otimes 1\right)\left(-|001\rangle + |010\rangle + |100\rangle + |111\rangle\right)$$

p = 1/4

001	010	100	111
GGR	GRG	RGG	RRR

Therefore, <u>always</u> an odd number of REDs R

Why ???

So where has the proof of impossibility gone awry?

The proof of impossibility is based on the following proposition:

<u>Proposition</u>: There exist no set of Boolean functions

$$f_A : \{0,1\} \to \{0,1\},\ f_B : \{0,1\} \to \{0,1\},\ f_B : \{0,1\} \to \{0,1\}$$

such that

$$f_A(s_1) + f_B(s_2) + f_C(s_3) = \begin{cases} 1(\bmod 2) & \textit{if} \quad s = 001, 010, 100 \\ 0(\bmod 2) & \textit{if} \quad s = 111 \end{cases}$$

The logic is flawless !!!

But the crux of the matter is that an argument is only as sound as the assumptions upon which it is based.

More explicitly, the argument fails because at least one of the following tacitly assumed two assumptions fails:

Premise 1. Reality Principle: What is measured is completely determined before it is measured.

Premise 2. Principle of Locality: Spacelike separated regions of spacetime are physically independent.

The above two premises lead to the following unfounded conclusions:

Unfounded Conclusion 1. Based on Premise 1 (The Reality Principle), the detector lamp instructions f_A, f_B, f_C must already be predetermined well-defined total functions at the time of particle ejection.

Unfounded Conclusion 2. Based on Premise 2 (The Principle of Locality), the detector lamp instructions f_A, f_B, f_C must be local. Hence, f_j is a function only of the j-th switch setting s_j and independent of the two other switch settings.

Corollary 1. For a switch settings $s = (s_1, s_2, s_3)$ of odd Hamming weight, the detector lamp instructions f_A, f_B, f_C are the random partial functions given by:

$$\begin{cases} f_A(s) = j_1 \\ f_B(s) = j_2 \\ f_C(s) = j_3 \end{cases}$$

with the Boolean algebraic dependence

$$f_A(s) + f_B(s) + f_C(s) = \sigma_2(s_1, s_2, s_3) + 1 \pmod 2$$

where σ_2 denotes the second elementary symmetric function

$$\sigma_2(s_1, s_2, s_3) = s_1 s_2 + s_2 s_3 + s_3 s_1$$

In other words, the GHZ paradox shows how to create three spacelike separated (hence, physically independent) probability distributions that have the above algebraic dependence.

The Boolean function σ_2 quantifies the nonlocality of the GHZ paradox.