CMSC 643
EXERCISES WITH BRAS ANSD KETS

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Let $\mathcal{H}$ be a Hilbert space with othonormal basis

$$\{|0\rangle, |1\rangle, |2\rangle, |3\rangle\} \ ,$$

and let $\mathcal{K}$ be a Hilbert space with othonormal basis

$$\{|a\rangle, |b\rangle, |c\rangle\}$$

(1) Represent each basis element of $\mathcal{H}$ as a column vector.
(2) Represent each basis element of $\mathcal{K}$ as a column vector
(3) Represent
$$|\psi\rangle = 2|0\rangle + 3i|2\rangle - 5|3\rangle$$
as a column vector
(4) Write $|1\rangle\langle 2|$ as a matrix
(5) Express
$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}$$
as a sum of $|i\rangle\langle j|$'s
(6) If
$$\begin{cases} |\psi_1\rangle = i|0\rangle - 2|2\rangle + 4|3\rangle \\ \\ |\psi_2\rangle = 2|0\rangle - 5|1\rangle - 7i|3\rangle \end{cases}$$
then compute $\langle\psi_1 \, |\psi_2\rangle$.

(7) Let $|\psi_1\rangle$ and $|\psi_2\rangle$ be as in (6) above. Express $|\psi_1\rangle\langle\psi_2|$
  (a) In terms of the bra's $\{\langle 0|, \langle 1|, \langle 2|, \langle 3|\}$ and the ket's $\{|0\rangle, |1\rangle, |2\rangle, |3\rangle\}$
  (b) As a matrix
(8) Let $|\psi_1\rangle$ and $|\psi_2\rangle$ be as in (6) above, and let
$$\begin{cases} |\varphi_1\rangle = -2|a\rangle - 3i|b\rangle + i|c\rangle \\ \\ |\varphi_2\rangle = 5|a\rangle + 7|b\rangle + 6i|c\rangle \end{cases}$$
Express
$$|\psi_1\rangle\langle\psi_2| \otimes |\varphi_1\rangle\langle\varphi_2|$$
as a 12×12 matrix.