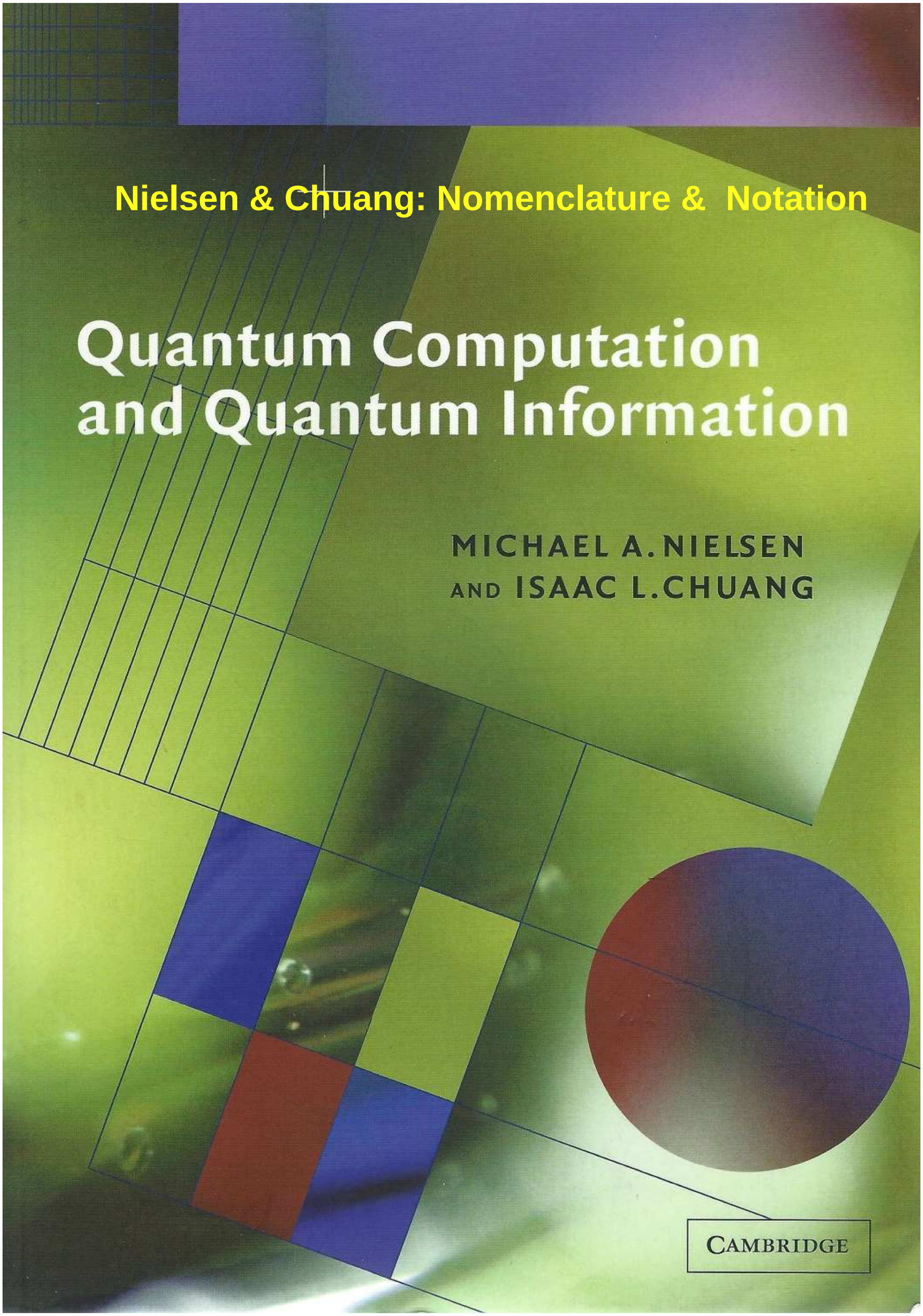
Nielsen & Chuang: Nomenclature & Notation



Nomenclature and notation

There are several items of nomenclature and notation which have two or more meanings in common use in the field of quantum computation and quantum information. To prevent confusion from arising, this section collects many of the more frequently used of these items, together with the conventions that will be adhered to in this book.

Linear algebra and quantum mechanics

All vector spaces are assumed to be finite dimensional, unless otherwise noted. In many instances this restriction is unnecessary, or can be removed with some additional technical work, but making the restriction globally makes the presentation more easily comprehensible, and doesn't detract much from many of the intended applications of the results.

A *positive* operator A is one for which $\langle\psi|A|\psi\rangle \geq 0$ for all $|\psi\rangle$. A *positive definite* operator A is one for which $\langle\psi|A|\psi\rangle > 0$ for all $|\psi\rangle \neq 0$. The *support* of an operator is defined to be the vector space orthogonal to its kernel. For a Hermitian operator, this means the vector space spanned by eigenvectors of the operator with non-zero eigenvalues.

The notation U (and often but not always V) will generically be used to denote a unitary operator or matrix. H is usually used to denote a quantum logic gate, the *Hadamard gate*, and sometimes to denote the *Hamiltonian* for a quantum system, with the meaning clear from context.

Vectors will sometimes be written in column format, as for example,

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \tag{0.1}$$

and sometimes for readability in the format $(1, 2)$. The latter should be understood as shorthand for a column vector. For two-level quantum systems used as qubits, we shall usually identify the state $|0\rangle$ with the vector $(1, 0)$, and similarly $|1\rangle$ with $(0, 1)$. We also define the Pauli sigma matrices in the conventional way – see 'Frequently used quantum gates and circuit symbols', below. Most significantly, the convention for the Pauli sigma z matrix is that $\sigma_z|0\rangle = |0\rangle$ and $\sigma_z|1\rangle = -|1\rangle$, which is reverse of what some physicists (but usually not computer scientists or mathematicians) intuitively expect. The origin of this dissonance is that the $+1$ eigenstate of σ_z is often identified by physicists with a so-called 'excited state', and it seems natural to many to identify this with $|1\rangle$, rather than with $|0\rangle$ as is done in this book. Our choice is made in order to be consistent with the usual indexing of matrix elements in linear algebra, which makes it natural to identify the first column of σ_z with the action of σ_z on $|0\rangle$, and the second column with the action on $|1\rangle$. This choice is also in use throughout the quantum computation and quantum information community. In addition to the conventional notations σ_x, σ_y and σ_z for the Pauli sigma matrices, it will also be convenient to use the notations $\sigma_1, \sigma_2, \sigma_3$ for these three matrices, and to define σ_0 as the 2×2 identity matrix. Most often, however, we use the notations I, X, Y and Z for $\sigma_0, \sigma_1, \sigma_2$ and σ_3, respectively.

Information theory and probability

As befits good information theorists, logarithms are *always* taken to base two, unless otherwise noted. We use $\log(x)$ to denote logarithms to base 2, and $\ln(x)$ on those rare occasions when we wish to take a natural logarithm. The term *probability distribution* is used to refer to a finite set of real numbers, p_x, such that $p_x \geq 0$ and $\sum_x p_x = 1$. The *relative entropy* of a positive operator A with respect to a positive operator B is defined by $S(A\|B) \equiv \mathrm{tr}(A \log A) - \mathrm{tr}(A \log B)$.

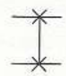
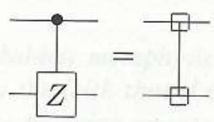
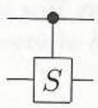
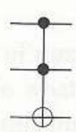
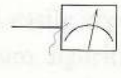
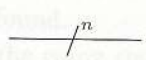
Miscellanea

$\oplus$ denotes modulo two addition. Throughout this book 'z' is pronounced 'zed'.

Frequently used quantum gates and circuit symbols

Certain schematic symbols are often used to denote unitary transforms which are useful in the design of quantum circuits. For the reader's convenience, many of these are gathered together below. The rows and columns of the unitary transforms are labeled from left to right and top to bottom as $00\ldots0, 00\ldots1$ to $11\ldots1$ with the bottom-most wire being the least significant bit. Note that $e^{i\pi/4}$ is the square root of i, so that the $\pi/8$ gate is the square root of the phase gate, which itself is the square root of the Pauli-Z gate.

Hadamard	$-\boxed{H}-$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Pauli-X	$-\boxed{X}-$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli-Y	$-\boxed{Y}-$	$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli-Z	$-\boxed{Z}-$	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Phase	$-\boxed{S}-$	$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
$\pi/8$	$-\boxed{T}-$	$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$

controlled-NOT		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
swap		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
controlled-Z		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
controlled-phase		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & i \end{bmatrix}$
Toffoli		$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$
Fredkin (controlled-swap)		$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$
measurement		Projection onto $\|0\rangle$ and $\|1\rangle$
qubit		wire carrying a single qubit (time goes left to right)
classical bit		wire carrying a single classical bit
n qubits		wire carrying n qubits