

**CMSC 442/653**  
**Instructor: Dr. Lomonaco**  
**Homework 6**

- **Listening assignment:** Listen to Brahms' Symphony No. 1
- **Reading Assignment:** The Study Problem Handout which can be downloaded by following the following link:  
<http://www.csee.umbc.edu/~lomonaco/f14/653/handouts/StudyProbs653.pdf>
- **Optional reading assignment:** Peterson, "Error-Correcting Codes," MIT Press, (1961), Chapters 8.

1) Consider the following degree 4 irreducible polynomial  $p(x)$  given in Peterson's Table of Irreducible Polynomials over  $\mathbf{GF}(2)$

**DEGREE 4 ... 3 37D ...**

- a) Write down  $p(x)$ .
- b) Since  $p(x)$  is irreducible and of degree 4, it follows that

$$\mathbf{GF}(2^4) = \mathbf{GF}(2)[x] \bmod p(x)$$

List all the elements of  $\mathbf{GF}(2^4)$  in the above representation, i.e., in terms of

$$\xi = x \bmod p(x)$$

- c) Let  $\xi = x \bmod p(x)$ . Why is  $\{\xi^k\}$  not a complete list of all the non-zero elements of  $\mathbf{GF}(2^4)$ ?

2) Let  $V$  be the cyclic code in  $R_{15} = \mathbf{GF}(2)[x]/(x^{15} + 1)$  given by the generator polynomial

$$g(x) = x^8 + x^4 + x^2 + x + 1.$$

- a) What is the length  $n$  of  $V$ ?
- b) What is the dimension  $k$  of  $V$ ?
- c) Use the generator polynomial  $g(x)$  to construct a generator matrix  $G$  for  $V$ .
- d) What is the parity check polynomial  $h(x)$  of  $V$ ?
- e) Use the parity check polynomial  $h(x)$  to construct the parity check matrix  $H$  of  $V$ .

3) Given that

$$x^9 + 1 = (x + 1)(x^2 + x + 1)(x^6 + x^3 + 1)$$

is a complete factorization over  $GF(2)$  of  $x^9 + 1$  into irreducible polynomials,

a) Draw the lattice of all ideals in  $R_9 = GF(2)[x]/(x^9 + 1)$ .

b) Determine the dimension of each ideal in  $R_9$ .

c) Determine the number of elements in each ideal in  $R_9$ .

d) List all the elements of the ideals

$$(x^6 + x^3 + 1) \text{ and } ((x + 1)(x^6 + x^3 + 1))$$