# CMSC 442/653
# Instructor: Dr. Lomonaco

## Homework 1

- **Listening Assignment:** Listen to Beethoven's 5-th symphony.
- **Reading Assignment:** http://www.cs.umbc.edu/~lomonaco/s06/652/slides/Equilateral-Triangle.pdf
- **Optional Reading assignment:** Peterson & Weldon, "Error-Correcting Codes," MIT Press, (Second Edition), Chapter 2.

**1)** Construct the multiplication table of the group of symmetries of the equilateral triangle given by the presentation

$$\left( \rho,\sigma : \rho^3 = 1, \sigma^2 = 1, \rho\sigma = \sigma\rho^2 \right)$$

Assume that the distinct group elements are:

$$1, \rho, \rho^2, \sigma, \rho\sigma, \rho^2\sigma$$

**2)** Construct the multiplication table of the group of symmetries of the square given by the presentation

$$\left( \rho,\sigma : \rho^4 = 1, \sigma^2 = 1, \rho\sigma = \sigma\rho^3 \right)$$

Assume that the distinct group elements are:

$$\left\{ \rho^m\sigma^n : 0 \le m < 4, 0 \le n < 2 \right\}$$

**3)** Let $S$ be a set with an associative binary operation $\bullet : S \times S \to S$. Let $e_L$ be a left identity of $S$ (i.e., $e_L \bullet s = s \; \forall s \in S$), and let $e_R$ be a right identity of $S$ (i.e., $s \bullet e_R = s \; \forall s \in S$).

a) Prove that $e_L = e_R$.

b) Also prove that $S$ can have at most one 2-sided identity.

**4)** Let $S$ be a set with an associative binary operation $\bullet : S \times S \to S$ and a 2-sided identity $e$, and let $s \in S$. Let $\tilde{s}_L$ and $\tilde{s}_R$ be elements of $S$ such that

$$\tilde{s}_L \bullet s = e = s \bullet \tilde{s}_R$$

Prove that $\tilde{s}_L = \tilde{s}_R$.