CMSC 442/653
Instructor: Dr. Lomonaco
Homework 8 ***CORRECTED***

- **Optional listening assignment:** Listen to Rachmanioff's Piano Concerto No. 4
- **Reading Assignment:** The Study Problem Handout which can be downloaded by following the following link:
  http://www.csee.umbc.edu/~lomonaco/f11/653/handouts/StudyProbs653.pdf
- **Optional reading assignment:** Peterson, "Error-Correcting Codes," MIT Press, (1961), Chapters 8.

1) Consider the following degree 4 irreducible polynomial **p(x)** given in Peterson's Table of Irreducible Polynomials over **GF(2)**

DEGREE 4 ... 3 37D ...

a) Write down **p(x)** .
b) Since p(x) is irreducible and of degree **4**, it follows that

$$GF(2^4) = GF(2)[x] \bmod p(x)$$

List all the elements of $GF(2^4)$ in the above representation, i.e., in terms of

$$\xi = x \bmod p(x)$$

c) Let $\xi = x \bmod p(x)$. Why is $\{\xi^k\}$ not a complete list of all the non-zero elements of $GF(2^4)$?

2) Let V be the cyclic code in $R_{15} = GF(2)[x]/\left(x^{15}+1\right)$ given by the generator polynomial

$$g(x) = x^8 + x^4 + x^2 + x + 1 \ .$$

a) What is the length n of V ?
b) What is the dimension k of V ?
c) Use the generator polynomial $g(x)$ to construct a generator matrix G for V .
d) What is the parity check polynomial $h(x)$ of V ?
e) Use the parity check polynomial $h(x)$ to construct the parity check matrix H of V .

3) Given that

$$x^9+1=(x+1)(x^2+x+1)(x^6+x^3+1)$$

is a complete factorization over $GF(2)$ of x^9+1 into irreducible polynomials,

a) Draw the lattice of all ideals in $R_9 = GF(2)[x]/(x^9+1)$.

b) Determine the dimension of each ideal in R_9.

c) Determine the number of elements in each ideal in R_9.

d) List all the elements of the ideals

$$(x^6+x^3+1) \text{ and } ((x+1)(x^6+x^3+1))$$