

CMSC 442/653
Instructor: Dr. Lomonaco
Homework 4

- **Listening Assignment:** Listen to Berlioz's Symphonie Fantastique
- **Optional Reading assignment:** Peterson & Weldon, "Error-Correcting Codes," MIT Press, (Second Edition), Chapters 2, 3, 6.

1) Let V be the linear code over $GF(3)$ determined by the generator matrix

$$G = \begin{pmatrix} 0 & 2 & 1 & 2 & 0 \\ 2 & 1 & 1 & 0 & 2 \\ 2 & 2 & 0 & 1 & 1 \end{pmatrix}$$

- What is the length n of V .
- Put the generator matrix of V in echelon canonical form to find the dimension k of V .
- Find a parity check matrix H for the linear code V .

2) Let V be the binary linear code given by the generator matrix

$$G = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

- Use the generator matrix to create a list of all code vectors of V .
- Use the list generated in a) to determine the minimum d distance of V .
- Explain why it is easier to find the minimum distance for a linear code than it is for a non-linear code.
- Find a parity check matrix H of the binary linear code V .

3) Let V be the binary linear code given by the parity check matrix

$$H = \begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

Find a generator matrix G for V .

4) Let V be a binary linear code given by the generator matrix

$$G = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

- a) Find a parity check matrix H of V .
- b) Use H to create an maximum likelihood error/syndrome table without first constructing the standard array.
- c) Demonstrate how your error/syndrome table can be used to decode the received vector $\mathbf{r} = 111101$.

5) Let V be the Hamming $[15,11] d = 3$ binary linear code.

- a) Write down the parity check matrix H .
- b) If

$$\vec{r} = 1000 \ 1000 \ 0000 \ 001$$

is a received vector, then what is the most likely error pattern. What is the most likely codevector that was originally sent? Please explain how you obtained your answers.