An Example of the Simplification of a Telescoping Product Expression

Problem: Simplify the following expression as much as possible:

$$P=\prod_{k=2}^{n}\frac{k\left(k+2\right)}{\left(k-1\right)\left(k+1\right)}$$

Answer:

$$P=\prod_{k=2}^{n}\frac{k\left(k+2\right)}{\left(k-1\right)\left(k+1\right)}=\frac{\prod_{k=2}^{n}k\left(k+2\right)}{\prod_{k=2}^{n}\left(k-1\right)\left(k+1\right)}$$

But under the change of variable $j=k-1\Leftrightarrow k=j+1$, we have

$$\prod_{k=2}^{n}\left(k-1\right)\left(k+1\right)=\prod_{j=2}^{n-1}j\left(j+2\right)=\prod_{k=2}^{n-1}k\left(k+2\right)$$

Hence,

$$P=\frac{\prod_{k=2}^{n}k\left(k+2\right)}{\prod_{k=1}^{n-1}k\left(k+2\right)}=\frac{n\left(n+1\right)\prod_{k=2}^{n-1}k\left(k+2\right)}{1\left(1+2\right)\prod_{k=2}^{n-1}k\left(k+2\right)}=\frac{n\left(n+2\right)}{3}$$