

CMSC 442/653
Fall 2006
Instructor: Dr. Lomonaco
Homework 7

- **Optional Reading assignment:** Peterson & Weldon, "Error-Correcting Codes," MIT Press, (Second Edition), Chapters 6 & 8
- **Optional Reading assignment:** MacWilliams & Sloane, "The Theory of Error-Correcting Codes," North-Holland (2nd edition), (1983), Chapter 7.

1U) Construct the addition and multiplication tables for the ring

$$R_4 = GF(2)[x]/(x^4 + 1)$$

2U) Let V be the cyclic code in $R_{15} = GF(2)[x]/(x^{15} + 1)$ given by the generator polynomial

$$g(x) = x^8 + x^4 + x^2 + x + 1 .$$

- a) What is the length n of V ?
- b) What is the dimension k of V ?
- c) Use the generator polynomial $g(x)$ to construct a generator matrix G for V .

3U) Given that

$$x^9 + 1 = (x + 1)(x^2 + x + 1)(x^6 + x^3 + 1)$$

is a complete factorization over $GF(2)$ of $x^9 + 1$ into irreducible polynomials,

- a) Draw the lattice of all ideals in $R_9 = GF(2)[x]/(x^9 + 1)$.
- b) Determine the dimension of each ideal in R_9 .
- c) Determine the number of elements in each ideal in R_9 .
- d) List all the elements of the ideals

$$(x^6 + x^3 + 1) \text{ and } ((x + 1)(x^6 + x^3 + 1))$$

4U) Given that

$$x^5 + 1 = (x + 1)(x^4 - x^3 + x^2 - x + 1)$$

is a complete factorization over $GF(3)$ of $x^5 + 1$ into irreducible polynomials,

- a) Draw the lattice of all ideals in $R_5^{(3)} = GF(3)[x]/(x^5 + 1)$
- b) Determine the dimension of each ideal in $R_5^{(3)}$.
- c) Determine the number of elements in each ideal in $R_5^{(3)}$.