CMSC 442/653
Fall 2006
Instructor: Dr. Lomonaco
Homework 5

- **Reading Assignment:** Review relevant slides on "Overview of Coding Theory" found at http://www.cs.umbc.edu/~lomonaco/f06/653/Slides653.html
- **Optional Reading assignment:** Peterson & Weldon, "Error-Correcting Codes," MIT Press, (Second Edition), Chapter 3, Sections 3.1, 3.2, 3.4

1U) Let V be the linear code over $GF(3)$ given by the row span of the matrix

$$G = \begin{pmatrix} 0 & 0 & 2 & 2 & 0 & 2 \\ 2 & 2 & 0 & 2 & 1 & 2 \\ 1 & 1 & 2 & 0 & 2 & 2 \\ 1 & 1 & 0 & 1 & 2 & 1 \end{pmatrix}$$

a) Put the matrix G into echelon canonical form.
b) Use the resulting echelon canonical form to find a basis for the linear code V. Explain your answer.
c) What is the dimension of V? Explain how you found your answer.

2U) Let V be a binary linear code given by the generator matrix

$$G = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

a) Find a parity check matrix H of V.
b) Construct a maximum likelihood decoding table for V.
c) Use H to reduce the maximum likelihood decoding table of **b)** to an error/syndrome table.
d) Demonstrate how your error/syndrome table can be used to decode the received vector

$$r = 111101$$

e) Use the generator matrix to create a list of all code vectors of V. Then use this list to determine the minimum distance of V.

3U) Let V be the binary linear code given by the parity check matrix

$$H = \begin{pmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

a) Construct an error/syndrome table for V without constructing the standard array.

b) Demonstrate how your error/syndrome table can be used to decode the received vector

$$r = 11100$$

4U) Let V be the Hamming $[15,11]$ $d = 3$ binary linear code.

a) Write down the parity check matrix H.

b) Find a generator matrix G for V. Please explain how you obtained your answer.

c) If

$$\vec{r} = 1000 \quad 1000 \quad 0000 \quad 001$$

is a received vector, then what is the most likely error pattern. Please explain how you obtained your answer.

5G) Prove that, if a binary linear code V has at least one vector of odd Hamming weight, then half the code vectors are of even Hamming weight, and the other half are of odd Hamming weight. You may assume without proof the following proposition:

Proposition. *Let V be a binary linear code, and let $H : V \to \mathbb{N}$ denote the Hamming weight function. If u and v are two arbitrary vectors in V, then*

a) $H((u+v))$ is even iff either both $H(u)$ and $H(v)$ are even or both $H(u)$ and $H(v)$ are odd.

b) $H((u+v))$ is odd iff either $H(u)$ is even and $H(v)$ is odd or $H(u)$ is odd and $H(v)$ is even.