CMSC 442/653
Fall 2006
Instructor: Dr. Lomonaco

Homework 1

- **Reading Assignment:** http://www.cs.umbc.edu/~lomonaco/s06/652/slides/Equilateral-Triangle.pdf

- **Optional Reading assignment:** Peterson & Weldon, "Error-Correcting Codes," MIT Press, (Second Edition), Chapter 2.

1) Construct the multiplication table of the group of symmetries of the equilateral triangle given by the presentation

$$\left(\rho,\sigma : \rho^3 = 1, \sigma^2 = 1, \rho\sigma = \sigma\rho^2 \right)$$

Assume that the distinct group elements are:

$$1, \rho, \rho^2, \sigma, \rho\sigma, \rho^2\sigma$$

2) Construct the multiplication table of the group of symmetries of the square given by the presentation

$$\left(\rho,\sigma : \rho^4 = 1, \sigma^2 = 1, \rho\sigma = \sigma\rho^3 \right)$$

Assume that the distinct group elements are:

$$\left\{ \rho^m\sigma^n : 0 \le m < 4, 0 \le n < 2 \right\}$$

Additional problem for grad students in CMSC 653:

Grad3) Let S be a set with an associative binary operation $\bullet : S \times S \to S$. Let e_L be a left identity of S (i.e., $e_L \bullet s = s \ \forall s \in S$), and let e_R be a right identity of S (i.e., $s \bullet e_R = s \ \forall s \in S$).

a) Prove that $e_L = e_R$.

b) Also prove that S can have at most one 2-sided identity.

Grad4) Let S be a set with an associative binary operation $\bullet : S \times S \to S$ and a 2-sided identity e, and let $s \in S$. Let $\widetilde{s_L}$ and $\widetilde{s_R}$ be elements of S such that

$$\widetilde{s_L} \bullet s = e = s \bullet \widetilde{s_R}$$

Prove that $\widetilde{s_L} = \widetilde{s_R}$.