CMSC 442 Fall 2003
Homework 5

- READING ASSIGNMENT: Peterson & Weldon, **"Error-Correcting Codes,"** MIT Press, (Second Edition), (1986), Chapter 5, section 5.1 to 5.5.

1) Let V be the the binary $(16, 11)$ $d = 4$ extended Hamming code.

- **a)** Find the parity check matrix H for V.
- **b)** Find the generator matrix G for V.
- **c)** Construct an error/syndrome table for V.
- **d)** Demonstrate the decoding procedure for the following two received vectotors:
  - 1010 1011 0000 0000
  - 1010 0000 0000 0000

2) Let V be the $(7, 4)$ $d = 3$ binary Hamming code. Devise a decoding procedure that corrects two or fewer erasures. **Hint:** If the codeword 1110000 is sent and $1x10y00$ is received, then only one of the four possible choices

$$(0,0)\,,(0,1)\,,(1,0)\,,(1,1)$$

for (x, y) will produce a codevector. Why?

3) **(Challenge Problem)** Let V be a binay linear code given by the parity check matrix

$$H = \begin{pmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_m \end{pmatrix}$$

where with linearly independent rows $\rho_1, \rho_2, \ldots, \rho_m$. Let V' be the augmented code formed from V given by the parity check matrix

$$H' = \begin{pmatrix} 0 \oplus \rho_1 \\ 0 \oplus \rho_2 \\ \vdots \\ 0 \oplus \rho_m \\ 1 \oplus \omega \end{pmatrix}$$

where $0 \oplus \rho_j$ denotes the vector $\left(0, \rho_j\right)$ for $j = 1, 2, \ldots, m$, where $1 \oplus \omega$ denotes the vector $(1, \omega)$, and where ω denotes the vector the all ones vector $(1, 1, \ldots, 1)$. Prove that the rows of H' are linearly independent.