

CMSC 442 Fall 2001
Instructor: Dr. Lomonaco
Homework 8

- **READING ASSIGNMENT:** Peterson & Weldon, “**Error-Correcting Codes,**” MIT Press, (Second Edition), (1986), Chapter 6 & 7
- **OPTIONAL READING ASSIGNMENT:** MacWilliams & Sloane, “**The Theory of Error-Correcting Codes,**” North-Holland (Second Edition), (1983), Chapter 7.

Prob. 1. You will find below the Antilog/Log table of $GF(2^6)$ based on the primitive polynomial

$$p(x) = x^6 + x + 1$$

Use this table to compute the minimum polynomial $m_5(x)$ of ξ^5 , where ξ is the primitive element defined by $p(x)$.

| Antilog $a_0a_1a_2a_3a_4a_5$ | Log | Antilog $a_0a_1a_2a_3a_4a_5$ | Log |
|---------------------------------|-----------|---------------------------------|-----|
| 000000 | $-\infty$ | 000101 | 15 |
| 100000 | 0 | 110010 | 16 |
| 010000 | 1 | 011001 | 17 |
| 001000 | 2 | 111100 | 18 |
| 000100 | 3 | 011110 | 19 |
| 000010 | 4 | 001111 | 20 |
| 000001 | 5 | 110111 | 21 |
| 110000 | 6 | 101011 | 22 |
| 011000 | 7 | 100101 | 23 |
| 001100 | 8 | 100010 | 24 |
| 000110 | 9 | 010001 | 25 |
| 000011 | 10 | 111000 | 26 |
| 110001 | 11 | 011100 | 27 |
| 101000 | 12 | 001110 | 28 |
| 010100 | 13 | 000111 | 29 |
| 001010 | 14 | 110011 | 30 |

| Antilog $a_0a_1a_2a_3a_4a_5$ | Log | Antilog $a_0a_1a_2a_3a_4a_5$ | Log |
|---------------------------------|-----|---------------------------------|-----|
| 101001 | 31 | 111001 | 47 |
| 100100 | 32 | 101100 | 48 |
| 010010 | 33 | 010110 | 49 |
| 001001 | 34 | 001011 | 50 |
| 110100 | 35 | 110101 | 51 |
| 011010 | 36 | 101010 | 52 |
| 001101 | 37 | 010101 | 53 |
| 110110 | 38 | 111010 | 54 |
| 011011 | 39 | 011101 | 55 |
| 111101 | 40 | 111110 | 56 |
| 101110 | 41 | 011111 | 57 |
| 010111 | 42 | 111111 | 58 |
| 111011 | 43 | 101111 | 59 |
| 101101 | 44 | 100111 | 60 |
| 100110 | 45 | 100011 | 61 |
| 010011 | 46 | 100001 | 62 |

Prob. 2. Consider $GF(3^3)$ defined by the primitive polynomial

$$p(x) = x^3 + 2x + 1,$$

and let $\xi = x \bmod p(x)$. Find the minimum polynomial $m_3(x)$ of ξ^3 . You may assume the following theorems:

$$a^p = a \text{ for all } a \in GF(p) \text{ for } p \text{ a prime integer.}$$

$$\left(\sum_{j=1}^n a^j \right)^p = \sum_{j=1}^n (a^j)^p \text{ in any field of characteristic } p.$$

You may use the following table for you calculations:

$GF(3^3)$ defined by the primitive polynomial $p(x) = x^3 + 2x + 1$

| Antilog $a_0a_1a_2$ | Log | Antilog $a_0a_1a_2$ | Log |
|-------------------------------|------------|-------------------------------|------------|
| 000 | $-\infty$ | | |
| 100 | 0 | 200 | 13 |
| 010 | 1 | 020 | 14 |
| 001 | 2 | 002 | 15 |
| 210 | 3 | 120 | 16 |
| 021 | 4 | 012 | 17 |
| 212 | 5 | 121 | 18 |
| 111 | 6 | 222 | 19 |
| 221 | 7 | 112 | 20 |
| 202 | 8 | 101 | 21 |
| 110 | 9 | 220 | 22 |
| 011 | 10 | 022 | 23 |
| 211 | 11 | 122 | 24 |
| 201 | 12 | 102 | 25 |

Prob. 3. a) Draw the linear sequential circuit (LSC) that multiplies by the polynomial

$$h(x) = 1 + x^3 + x^6$$

b) Draw the linear sequential circuit (LSC) that divides by the polynomial

$$g(x) = 1 + x^2 + x^4 + x^6 + x^7$$

c) Draw the linear sequential circuit (LSC) that simultaneously multiplies by $h(x)$ and divides by $g(x)$.

Prob. 4. Draw an LSC which takes as inputs polynomials $a(x)$ and $b(x)$ and then produces the output

$$h(x)a(x) + k(x)b(x),$$

where $h(x)$ and $k(x)$ are the polynomials:

$$h(x) = 1 + x^4 + x^{10} \text{ and } k(x) = x + x^2 + x^4 + x^7 + x^9$$