CMSC 442 Fall 2001
Instructor: Dr. Lomonaco
Homework 6

- READING ASSIGNMENT: Peterson & Weldon, **"Error-Correcting Codes,"** MIT Press, (Second Edition), (1986), Chapter 6 & 8
- OPTIONAL READING ASSIGNMENT: MacWilliams & Sloane, **"The Theory of Error-Correcting Codes,"** North-Holland (Second Edition), (1983), Chapter 7.

Problem 1. Given that

$$x^9 + 1 = (x+1)\left(x^2+x+1\right)\left(x^6+x^3+1\right)$$

is a complete factorization over $GF(2)$ of x^9+1 into irreducible polynomials,

a) Draw the lattice of all ideals in $R_9 = GF(2)[x]/ < x^9 + 1 >$

b) Determine the dimension of and the number of elements in each ideal of R_9.

c) List all elements of the ideals

$$\left\langle x^6+x^3+1\right\rangle \text{ and } \left\langle (x+1)\left(x^6+x^3+1\right)\right\rangle$$

Problem 2. Given that

$$x^5 + 1 = (x+1)\left(x^4 - x^3 + x^2 - x + 1\right)$$

is a complete factorization over $GF(3)$ of x^5+1 into irreducible polynomials,

a) Draw the lattice of all ideals in $R_9^{(3)} = GF(3)[x]/ < x^5 + 1 >$

b) Determine the dimension of and the number of elements in each ideal of $R_9^{(3)}$.

Problem 3. Let V denote the minimum length binary cyclic code given by the generator polynomial:

$$g(x) = x^8 + x^4 + x^2 + x + 1 \ .$$

a) Determine the lenght n of V

b) Use $g(x)$ to compute the generator matrix G of V. What is the dimension of V ?

c) Compute the generator polynomial $h^*(x)$ of $V^!$.

d) Use $h^*(x)$ to compute a generator matrix $H_!$ of $V^!$.

e) From $h^*(x)$, compute the parity check polynomial $h(x)$ of V.

f) Use the parity check polynomial $h(x)$ computed in e) to compute the parity check matrix H of V.