

**CMSC 442 Fall 2001**  
**Instructor: Dr. Lomonaco**  
**Homework 4**

- **READING ASSIGNMENT:** Peterson & Weldon, “**Error-Correcting Codes,**” MIT Press, (Second Edition), (1986), Chapter 6 & 8
- **OPTIONAL READING ASSIGNMENT:** MacWilliams & Sloane, “**The Theory of Error-Correcting Codes,**” North-Holland (Second Edition), (1983), Chapter 7.

**Problem 1.** Let  $V$  be the binary  $[15, 11, 3]$  Hamming code.

- a) Compute the weight enumerator of the null space  $V^\perp$  of  $V$ .
- b) Then use the MacWilliams identity to compute the weight enumerator of  $V$ .

**Problem 2.** Compute the addition and multiplication tables for the ring

$$R_4 = GF(2)[x]/\langle x^4 + 1 \rangle$$

Also express each of the following ideals in the ring  $R_4$  as a set of polynomials in  $R_4$  of degree less than 4.

$$\begin{aligned} &\langle 0 \rangle, \langle 1 + x \rangle, \langle 1 + x + x^2 \rangle, \langle 1 + x^2 + x^3 \rangle, \langle 1 + x + x^2 + x^3 \rangle, \\ &\langle 1 + x^2 \rangle, \langle 1 + x^3 \rangle, \langle 1 + x + x^5 \rangle, \langle x + x^3 \rangle, \langle x^6 + x^{10} + x^{123} \rangle. \end{aligned}$$

For example,

$$\langle 0 \rangle = \{0\} \text{ and } \langle 1 + x^5 + x^6 + x^{19} \rangle = \{0, 1 + x + x^2 + x^3\}$$