CMSC 442 Fall 2001
Homework 1

Due: Monday, September 17, 2001 no later than 6pm.

- Reading assignment: Peterson & Weldon, **"Error-Correcting Codes,"** MIT Press, (Second Edition), (1986), Chapters 1 & 2.
- Optional reading assignment: MacWilliams & Sloane, **"The Theory of Error-Correcting Codes,"** North-Holland (Second Edition), (1983), Chapter 1.

1. Form a maximum-likelihood decoding table fot the binary code consisting of the four code words 0000, 0011, 1100, and 1111, assuming the binary symmetric channel.

2. A "metric" function is defined as a real-valued function having the following three properties:

   (a) $d(x, y) = 0$ iff $x = y$ (Reflexivity)

   (b) $d(x, y) = d(y, x)$ (Symmetry)

   (c) $d(x, y) \leq d(y, z) + d(x, z)$ (Triangle Inequality)

   Show that the Hamming distance is a metric function.

   Hint. $H(x, y) = Wt(x \oplus y)$, where $H(x, y)$ denotes the Hamming distance between x and y, where $Wt(v)$ denotes the Hamming weight of v (i.e., the number of 1's in v, and where '$\oplus$' denotes componentwise addition mod 2.

3. Show that the minimum Hamming distance of at least $2t + 1$ between code blocks is necessary and sufficient for correcting all combinations of t or fewer errors. **Hint.** Use the triangle inequality.

4. Construct the multiplication table of the group G given by the presentation:

   $$\left(\rho, \sigma : \rho^4 = \sigma^2 = 1, \sigma\rho = \rho^3\sigma\right)$$

   You may assume that the distinct group elements are:

   $$1, \rho, \rho^2, \rho^3, \sigma, \rho\sigma, \rho^2\sigma, \rho^3\sigma$$

   Also compute in the ring $GF(2)G$ the product

   $$(1 + \rho^3 + \rho^2\sigma)(\rho + \sigma + \rho^3\sigma)$$