

A Simple but Illustrative Example of a Quantum Error Correcting Code

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Part I: Classical Coding Theory

A Simple But Illustrative Example
of
Classical Coding Theory

A Very, Very, ..., Very Simple Error-Correcting Code 119ECC
CS

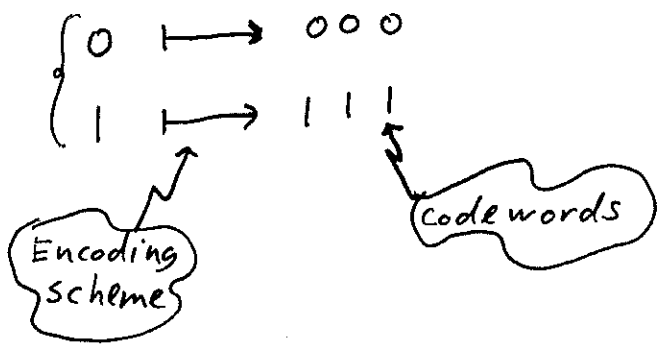
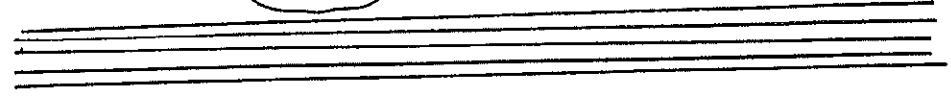
3-Repeat Code (Hamming (3,1) code)

$$V = \{000, 111\} \subseteq GF(2)^3$$

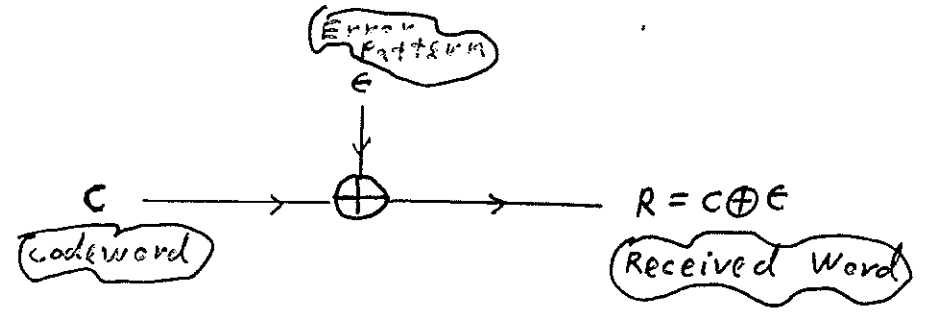
V is the row space of the matrix

$$G = [111]$$

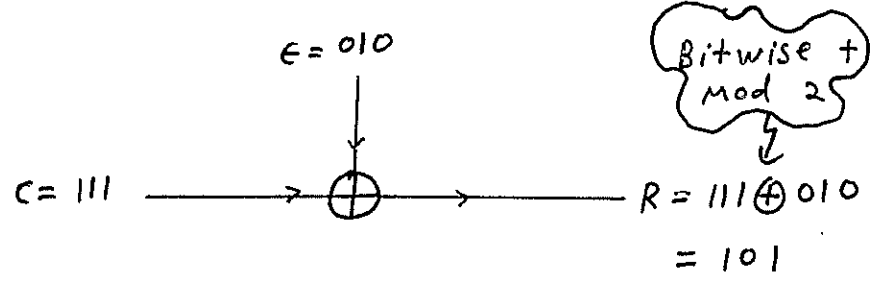
Generator Matrix of V



Model of Communication Channel 129ECC
CS



For example,



Syndrome Table for 3-Repeat Code

159810
c9

Possible Error Pattern	Syndrome
000	00
001	11
010	10
100	01
011	01
101	10
110	11
111	00

Syndrome Table for 3-Repeat Code

169810
c9

Possible Error Pattern	Syndrome	
Most Likely Errors {	000	00
	001	11
	010	10
	100	01
Most Likely Errors {	011	01
	101	10
	110	11
	111	00

Principle: Maximum Likelihood Decoding

Error Correction Procedure:

Correct Most Likely Error Patterns

Part I: QECCs

A Simple But Illustrative

Example Of

Quantum Error Correcting Codes

Bits Represented as Quantum States 1998
CIT

$$\left\{ \begin{array}{l} 0 \leftrightarrow |0\rangle = 1 \cdot |0\rangle + 0 \cdot |1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ 1 \leftrightarrow |1\rangle = 0 \cdot |0\rangle + 1 \cdot |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{array} \right.$$

Observing Parity

12
49EE
C17

Consider observable

$$O = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Therefore,

$$\begin{cases} O|0\rangle = |0\rangle \\ O|1\rangle = -|1\rangle \end{cases}$$

eigenvalue	Generators of Eigen space
+1	$ 0\rangle$
-1	$ 1\rangle$

Observing Parity Checks

11
59EE
C18

$$H = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \leftrightarrow I \otimes 0 \otimes 0 \\ \leftrightarrow 0 \otimes I \otimes 0$$

Observable ~~0000~~ $I \otimes 0 \otimes 0$

Eigenvalue	Generators of Eigen space	EigenSpace	Dim	Parity Check 011 satisfied?
+1	$ 000\rangle, 100\rangle, 011\rangle, 111\rangle$	V_+^{011}	4	YES
-1	$ 001\rangle, 101\rangle, 010\rangle, 110\rangle$	V_-^{011}	4	No

Observable $0 \otimes I \otimes 0$

Eigenvalue	Generators of Eigen Space	EigenSpace	Dim	Parity Check 101 satisfied?
+1	$ 000\rangle, 010\rangle, 111\rangle, 101\rangle$	V_+^{101}	4	YES
-1	$ 001\rangle, 011\rangle, 100\rangle, 110\rangle$	V_-^{101}	4	NO

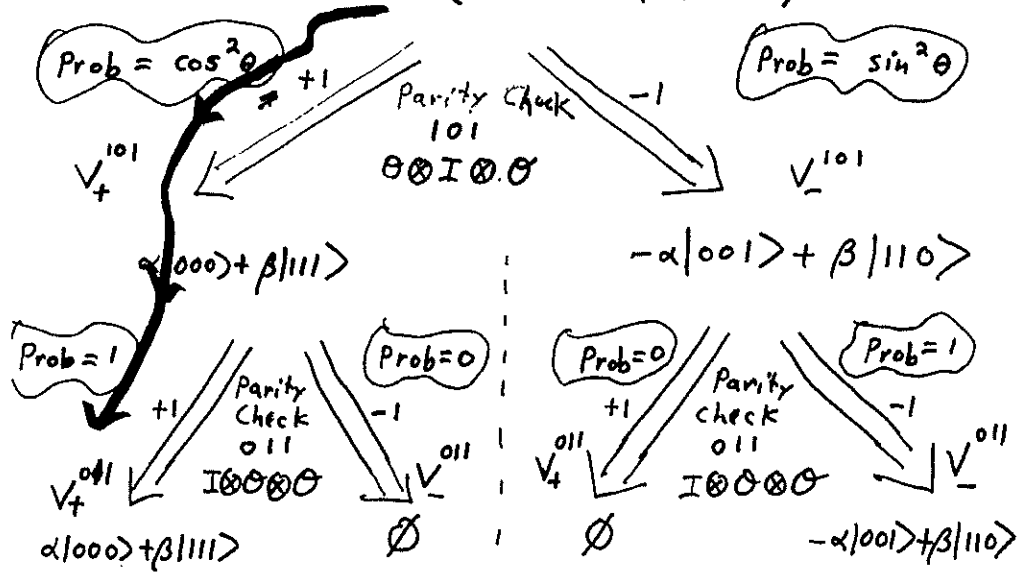
Syndrome Calculation

15
60/73
C19

$$\alpha|000\rangle + \beta|111\rangle$$

$$I \otimes I \otimes U_{\theta} \downarrow \quad \epsilon = 001$$

$$\cos\theta(\alpha|000\rangle + \beta|111\rangle) + \sin\theta(-\alpha|001\rangle + \beta|110\rangle)$$



\therefore syndrome is 00

$\epsilon = 000$ (Most Likely)

\therefore syndrome is 11

$\epsilon = 001$ (Most Likely)

A Q.M. Error Corrector

16
79ECC
C20

Error Corrector $C = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ Unitary Transf.

$$\begin{cases} C|0\rangle = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle \\ C|1\rangle = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} = -|0\rangle \end{cases}$$