Quantum Entanglement

Qubits

00

$$\left(\frac{|0\rangle + i|1\rangle}{\sqrt{2}}\right) \otimes \left(-i|1\rangle\right)$$

- Not Entangled
- Separate

EPR Pair

$$\frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

Einstein

Podolsky

Rosen

Bah ¡ Humbug ¡

Something is missing from Quantum Mechanics.

$\exists$ Hidden Variables

EPR Pair

$$\frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

~~Einstein~~

~~Podolsky~~

~~Rosen~~

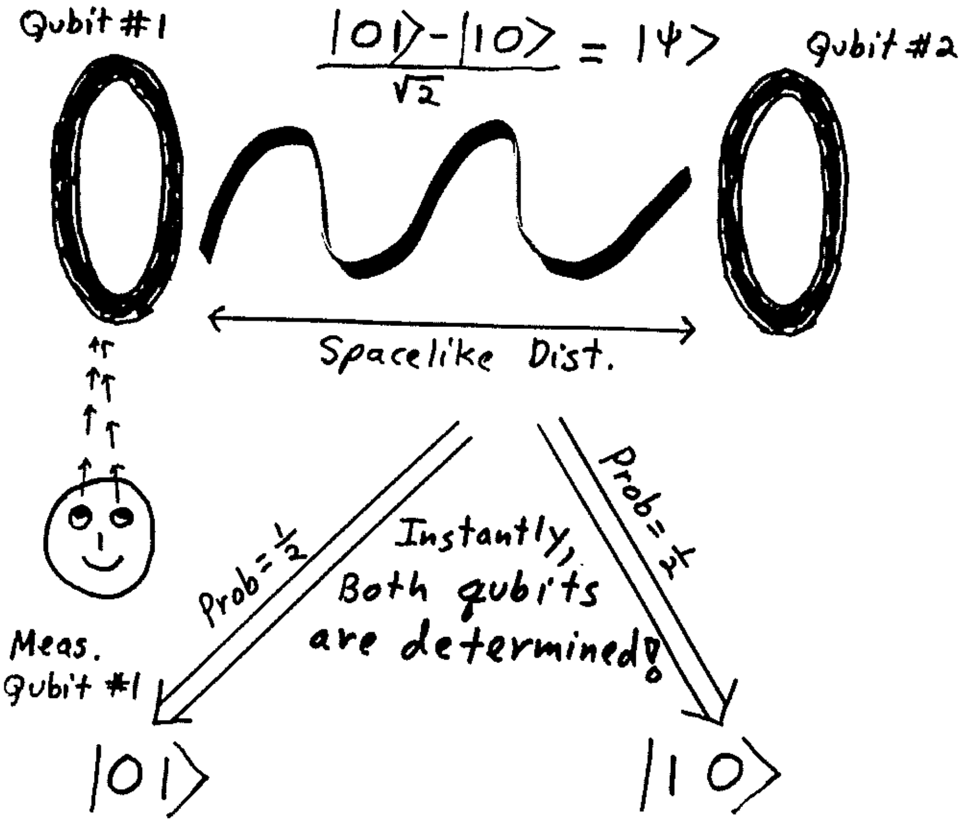
~~Bah ¡ Humbug ¡~~

~~Something is missing from Quantum Mechanics.~~

$\exists$ Hidden Variables

Bell Inequality

Measurement of EPR Pair

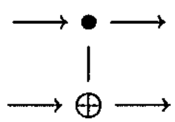


Non-Local Interaction!

- No force of any kind
  - Not mediated by anything
- Acts instantaneously
  - Faster than light
- Strength does not drop off with distance

Yet, still consistent with Gen. Relativity!

CNOT As An "Entangler"

INPUT Not Entangled		OUTPUT Entangled
$a\|0\rangle + b\|1\rangle$ $\|0\rangle$		$a\|00\rangle + b\|11\rangle$

Quantum Teleporting Manual

Step 1(Loc. A): Preparation

- At location A, construct an EPR pair of qubits (qubits #2 & #3) in $\mathcal{H}_2 \otimes \mathcal{H}_3$.

$$|00\rangle \longmapsto \boxed{\text{Unitary Matrix}} \longmapsto \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

$$\mathcal{H}_2 \otimes \mathcal{H}_3 \longrightarrow \mathcal{H}_2 \otimes \mathcal{H}_3$$

- Physically transport entangled qubit #3 from Loc. A to Loc. B

Result

- Loc. A & Loc. B share an EPR pair, i.e.,
  - Qubit #2 is at Loc. A
  - Qubit #3 is at Loc. B
  - Qubits #2 & #3 are entangled

- The state of all three qubits is:

$$|\Phi\rangle = (a|0\rangle + b|1\rangle)\left(\frac{|01\rangle - |10\rangle}{\sqrt{2}}\right) \in \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3.$$

Quantum Teleporting Manual (Cont.)

Step 2. (Loc. A): Apply $U \otimes I : \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3 \longrightarrow \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3$ to the three qubits. Thus, under $U \otimes I$ the state $|\Phi\rangle$ of all three qubits becomes:

$$
\begin{aligned}
|\Phi\rangle = \tfrac{1}{2}[\quad & |00\rangle\left(-a|0\rangle - b|1\rangle\right) \\
& + |01\rangle\left(-a|0\rangle + b|1\rangle\right) \\
& + |10\rangle\left(a|1\rangle + b|0\rangle\right) \\
& + |11\rangle\left(a|1\rangle - b|0\rangle\right) \quad]
\end{aligned}
$$

Step 3. (Loc. A): Measure qubits #1 & #2.

Step 4. (Loc. A): Send via a classical communication channel the result of the measurement to Loc. B.

Result

Unknown qubit #1 has been disassembled and the info read (two classical bits) is sent to Loc. B.

The Bell Basis

Let $\mathcal{H}_2$ be a 2-D Hilbert space with orthonormal basis

$$\{|0\rangle, |1\rangle\}$$

Then

$$\mathcal{H} = \mathcal{H}_2 \otimes \mathcal{H}_2$$

is a $2 \cdot 2 = 4$ dim'l Hilbert space with induced orthonormal basis

$$|00\rangle, |01\rangle, |10\rangle, |11\rangle$$

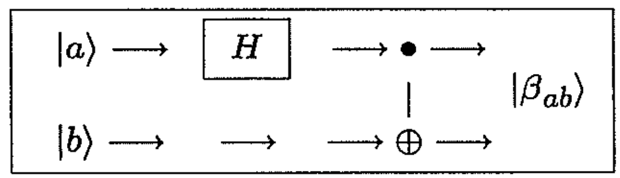
called the **standard basis**.

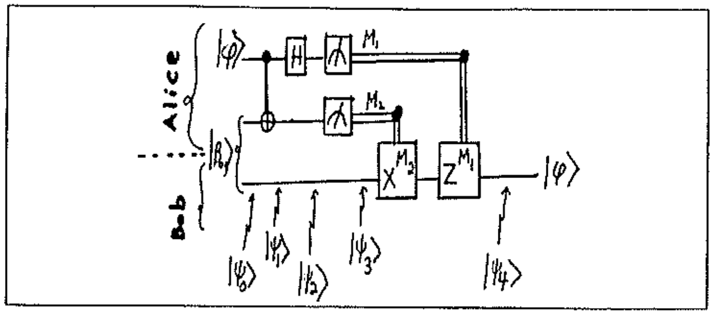
Another orthonormal basis is the **Bell basis**

$$\begin{cases} |\beta_{00}\rangle &= (I \otimes I)\,|\beta_{00}\rangle &= (|00\rangle + |11\rangle)/\sqrt{2} \\ |\beta_{01}\rangle &= (X \otimes I)\,|\beta_{00}\rangle &= (|10\rangle + |01\rangle)/\sqrt{2} \\ |\beta_{10}\rangle &= (Z \otimes I)\,|\beta_{00}\rangle &= (|00\rangle - |11\rangle)/\sqrt{2} \\ |\beta_{11}\rangle &= (ZX \otimes I)\,|\beta_{00}\rangle &= (-|10\rangle + |01\rangle)/\sqrt{2} \end{cases}$$

$$|\beta_{ab}\rangle = \left(Z^a X^b \otimes I\right)|\beta_{00}\rangle$$

The standard basis can be transformed into the Bell basis with the following unitary transformation



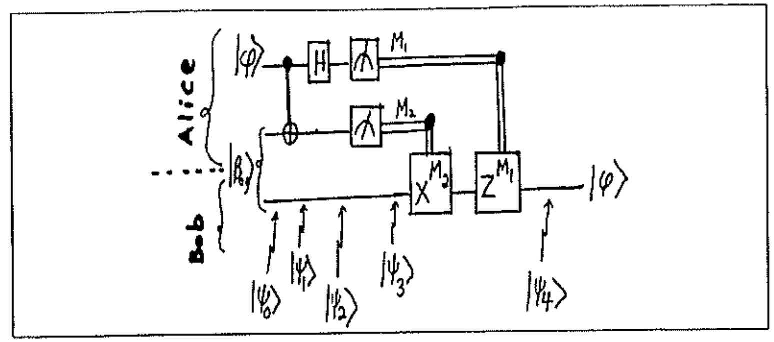


The initial state is

$$\boxed{|\psi_0\rangle = |\varphi\rangle\,|\beta_{00}\rangle}$$

After the CNOT, we have

$$\boxed{|\psi_1\rangle = \frac{1}{\sqrt{2}}\left[a\,|0\rangle\,(|00\rangle + |11\rangle) + b\,|1\rangle\,(|00\rangle + |01\rangle)\right]}$$



And after the Hadamard operation, we have

$$\begin{aligned}|\psi_2\rangle = \tfrac{1}{2}[\quad & |00\rangle\,(a\,|0\rangle + b\,|1\rangle) \\ + \; & |01\rangle\,(a\,|1\rangle + b\,|0\rangle) \\ + \; & |10\rangle\,(a\,|0\rangle - b\,|1\rangle) \\ + \; & |11\rangle\,(a\,|1\rangle - b\,|0\rangle)] \\ = \; & \sum_{M_1,M_2=0}^{1} |M_2 M_1\rangle\, X^{M_1} Z^{M_2}\,|\varphi\rangle\end{aligned}$$