## CMSC 461, Database Management Systems Spring 2018

## Lecture 14 - Chapter 8 Relational Database Design Wrap-up

These slides are based on "Database System Concepts" $6^{\text {th }}$ edition book and are a modified version of the slides which accompany the book
(http://codex.cs.yale.edu/avi/db-book/db6/slide-dir/index.html), in addition to the 2009/2012 CMSC 461 slides by Dr. Kalpakis

## Logistics

- Homework \#3 due today $3 / 26 / 2018$
- Phase 3 of project due $3 / 28 / 2018$


## Lecture Outline

- An Example
- Functional Dependencies Review
- BCNF Decomposition
- 3NF Decomposition
- Multivalued Decomposition
- Fourth Normal Form
- Database Design Process


## An Example

CREATE TABLE film_actor_nn ( film_actor_id BIGINT NOT NULL PRIMARY KEY, first_name VARCHAR(50) NOT NULL, last_name VARCHAR(50) NOT NULL, film_name varchar(100), producer varchar(50));

## An Example

insert into film_actor_nn values (1,'Michael', 'Douglas', 'The American President', 'Rob Reiner'); insert into film_actor_nn values (2,'Michael', 'Douglas', 'BettleJuice','Larry Wilson'); insert into film_actor_nn values (3,'Michael', 'Douglas', 'Fatal Attraction','Stanley R. Jaffe');

## An Example

SELECT film_actor_id, first_name, last_name, COUNT(*) FROM film_actor_nn GROUP BY film_actor_id ORDER BY COUNT(*) DESC;

| film_actor_id | first_name | last_name | COUNT(*)|


| $2 \mid$ Michael | $\mid$ Douglas $\mid$ |
| :--- | :--- |
| $3 \mid$ Michael | Douglas $\mid$ |
| $\mid$ | $1 \mid$ |
| $1 \mid$ Michael | Douglas $\mid$ |
| $\mid$ | $1 \mid$ |



3 rows in set ( 0.00 sec )

## An Example

CREATE TABLE film_actor_nn ( film_actor_id BIGINT NOT NULL PRIMARY KEY, first_name VARCHAR(50) NOT NULL, last_name VARCHAR(50) NOT NULL,<br>film_name varchar(100), producer varchar(50));

## An Example

CREATE TABLE actor ( actor_id BIGINT NOT NULL PRIMARY KEY, first_name VARCHAR(50) NOT NULL, last_name VARCHAR(50) NOT NULL);

CREATE TABLE film (film_id BIGINT NOT NULL PRIMARY KEY, film_name VARCHAR(50) NOT NULL, producer VARCHAR(50) NOT NULL );

CREATE TABLE film_actor (actor_id BIGINT NOT NULL REFERENCES actor(actor_id), film_id BIGINT NOT NULL references film(film_id), PRIMARY KEY (actor_id, film_id));

## An Example

insert into actor values (1, 'Michael', 'Douglas'); insert into actor values (2, 'Michael', 'Douglas');
insert into film values (100, 'The American
President','Rob Reiner'); insert into film values (200, 'BettleJuice','Larry Wilson');
insert into film_actor values (1,100); insert into film_actor values $(2,200)$;

## An Example

SELECT actor_id, first_name, last_name, COUNT(*) FROM actor JOIN film_actor USING (actor_id) GROUP BY actor_id ORDER BY COUNT(*) DESC;
+----------------------+----------------------
| actor_id | first_name | last_name | COUNT(*) |
+----------------------+---------------------+

| 1 \| Michael | Douglas | 1 |
| :---: | :---: | :---: |
| 2 \| Michael | Douglas | 1 |

## An Example

insert into film values (300, 'Fatal Attraction','Stanley R.
Jaffe');
insert into film_actor values $(1,300)$;

## An Example

SELECT actor_id, first_name, last_name, COUNT(*) FROM actor JOIN film_actor USING (actor_id) GROUP BY actor_id ORDER BY COUNT(*) DESC;
+----------------------+----------------------
| actor_id | first_name | last_name | COUNT(*) |
+-------------------------------------------+

| 1 \| Michael | Douglas | 2 |
| :---: | :---: | :---: |
| 2 \| Michael | Douglas | 1 |

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## Closure of a Set of Functional

## Dependencies

- Given a set F of functional dependencies, there are certain other functional dependencies that are logically implied by $F$
- For example:

Given a schema r(A,B,C)
If $A \rightarrow B$ and $B \rightarrow C$
then we can infer that $A \rightarrow C$

- The set of all functional dependencies logically implied by $F$ is the closure of $F$
- We denote the closure of $F$ by $F^{+}$
- $F^{+}$is a superset of $F$


## Closure of a Set of Functional Dependencies

- We can find $\mathrm{F}^{+}$, the closure of F , by repeatedly applying Armstrong's Axioms:
- if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$
(reflexivity)
- if $\alpha \rightarrow \beta$, then $\gamma \alpha \rightarrow \gamma \beta$
(augmentation)
- if $\alpha \rightarrow \beta$, and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$
- These rules are
- sound (generate only functional dependencies that actually hold), and
- complete (generate all functional dependencies that hold).


## Computing F+

- To compute the closure of a set of functional dependencies F:
$F+=F$
repeat
for each functional dependency $f$ in $F^{+}$ apply reflexivity and augmentation rules on $f$ add the resulting functional dependencies to $F^{+}$
for each pair of functional dependencies $f_{1}$ and $f_{2}$ in $F+$ if $f_{1}$ and $f_{2}$ can be combined using transitivity then add the resulting functional dependency to $F+$
until $F+$ does not change any further


## Closure of a set of Functional Dependencies

- Additional rules:
- If $\alpha \rightarrow \beta$ holds and $\alpha \rightarrow \gamma$ holds, then $\alpha \rightarrow \beta$ $\gamma$ holds (union)
- If $\alpha \rightarrow \beta \gamma$ holds, then $\alpha \rightarrow \beta$ holds and $\alpha \rightarrow$ $\gamma$ holds (decomposition)
- If $\alpha \rightarrow \beta$ holds and $\gamma \beta \rightarrow \delta$ holds, then $\alpha \gamma$ $\rightarrow \delta$ holds (pseudotransitivity)
The above rules can be inferred from Armstrong's axioms.


## Closure of a set of Functional Dependencies Example

- $R=(A, B, C, G, H, I)$
$F=\{A \rightarrow B$
$A \rightarrow C$
$C G \rightarrow H$
$C G \rightarrow I$
$B \rightarrow H\}$
- some members of $F^{+}$
- $A \rightarrow H$
- by transitivity from $A \rightarrow B$ and $B \rightarrow H$
- AG $\rightarrow$ I
- by augmenting $A \rightarrow C$ with $G$, to get $A G \rightarrow C G$ and then transitivity with $C G \rightarrow I$
- $\mathrm{CG} \rightarrow \mathrm{HI}$
- by union rule, since $C G \rightarrow H$ and $C G \rightarrow I$, implies $C G \rightarrow H I$


## Closure of Attribute Sets

- Given a set of attributes $\alpha$, define the closure of $\alpha$ under $F$ (denoted by $\alpha^{+}$) as the set of attributes that are functionally determined by $\alpha$ under $F$
- Algorithm to compute $\alpha^{+}$, the closure of $\alpha$ under $F$

```
result := \(\alpha\);
while (changes to result) do
    for each \(\beta \rightarrow \gamma\) in \(F\) do
    begin
    if \(\beta \subseteq\) result then result \(:=\) result \(\cup \gamma\)
    end
```


## Closure of Attribute Sets Example

- $R=(A, B, C, G, H, I)$
- $F=\{A \rightarrow B$
$A \rightarrow C$
$C G \rightarrow H$
$C G \rightarrow I$
$B \rightarrow H\}$
- (AG)+
1.result = AG
2.result $=A B C G(A \rightarrow C$ and $A \rightarrow B)$
3.result $=A B C G H \quad(C G \rightarrow H$ and $C G \subseteq A G B C)$
4.result $=A B C G H I \quad(C G \rightarrow I$ and $C G \subseteq A G B C H)$


## Closure of Attribute Sets Uses

There are several uses of the attribute closure algorithm:

- Testing for superkey:
- To test if $\alpha$ is a superkey, we compute $\alpha^{+}$, and check if $\alpha^{+}$ contains all attributes of $R$.
- Testing functional dependencies
- To check if a functional dependency $\alpha \rightarrow \beta$ holds (or, in other words, is in $F^{+}$), just check if $\beta \subseteq \alpha^{+}$.
- That is, we compute $\alpha$ +by using attribute closure, and then check if it contains $\beta$.
- Is a simple and cheap test, and very useful
- Computing closure of F
- For each $\gamma \subseteq R$, we find the closure $\gamma^{+}$, and for each $S \subseteq \gamma^{+}$, we output a functional dependency $\gamma \rightarrow S$.


## Closure of Attribute Sets Examples

- $R=$ (name,color,category,department,price)
- $F=$ \{name $\rightarrow$ color category $\rightarrow$ department color,category $\rightarrow$ price $\}$

You find:

- name ${ }^{+}$
- \{name,category ${ }^{+}$
- \{color\}+


## Lossless-join Decomposition

- For the case of $R=\left(R_{1}, R_{2}\right)$, we require that for all possible relations $r$ on schema $R$

$$
r=\Pi_{R 1}\left(\kappa_{A}\right) \quad \Pi_{R 2}(r)
$$

- A decomposition of $R$ into $R_{1}$ and $R_{2}$ is lossless join if at least one of the following dependencies is in $F^{+}$:
- $R_{1} \cap R_{2} \rightarrow R_{1}$
- $R_{1} \cap R_{2} \rightarrow R_{2}$
- The above functional dependencies are a sufficient condition for lossless join decomposition; the dependencies are a necessary condition only if all constraints are functional dependencies


## Example

- $R=(A, B, C)$
$F=\{A \rightarrow B, B \rightarrow C\}$
- Can be decomposed in two different ways
- $R_{1}=(A, B), R_{2}=(B, C)$
- Lossless-join decomposition:

$$
R_{1} \cap R_{2}=\{B\} \text { and } B \rightarrow B C
$$

- Dependency preserving
- $R_{1}=(A, B), \quad R_{2}=(A, C)$
- Lossless-join decomposition:

$$
R_{1} \cap R_{2}=\{A\} \text { and } A \rightarrow \mathrm{AB}
$$

- Not dependency preserving
(cannot check $B \rightarrow C$ without computing $R_{1} \bowtie R_{2}$ )


## Dependency Preservation

- Let $F_{i}$ be the set of dependencies $F$ + that include only attributes in $R_{i}$.
- A decomposition is dependency preserving, if $\left(F_{1} \cup F_{2} \cup \ldots \cup F_{n}\right)^{+}=F^{+}$
- If it is not, then checking updates for violation of functional dependencies may require computing joins, which is expensive.


## Testing for Dependency Preservation

- To check if a dependency $\alpha \rightarrow \beta$ is preserved in a decomposition of $R$ into $R_{1}, R_{2}, \ldots, R_{\mathrm{n}}$ we apply the following test (with attribute closure done with respect to $F$ )
- result = $\alpha$
while (changes to result) do
for each $R_{i}$ in the decomposition

```
t=(result \cap R i}\mp@subsup{)}{}{+}\capR\mp@subsup{R}{i}{
result = result \cupt
```

- If result contains all attributes in $\beta$, then the functional dependency $\alpha \rightarrow \beta$ is preserved.


## Testing for Dependency Preservation

- We apply the test on all dependencies in $F$ to check if a decomposition is dependency preserving
- This procedure takes polynomial time, instead of the exponential time required to compute $F^{+}$and $\left(F_{1} \cup F_{2} \cup \ldots \cup F_{n}\right)^{+}$


## Testing for BCNF

- To check if a non-trivial dependency $\alpha \rightarrow \beta$ causes a violation of BCNF

1. compute $\alpha^{+}$(the attribute closure of $\alpha$ ), and
2. verify that it includes all attributes of $R$, that is, it is a superkey of $R$.

## Testing for BCNF

- Simplified test: To check if a relation schema $R$ is in BCNF, it suffices to check only the dependencies in the given set $F$ for violation of BCNF, rather than checking all dependencies in $F^{+}$.
- If none of the dependencies in $F$ causes a violation of BCNF, then none of the dependencies in $F^{+}$will cause a violation of BCNF either.


## Testing for BCNF

- However, simplified test using only $F$ is incorrect when testing a relation in a decomposition of $R$
- Consider $R=(A, B, C, D, E)$, with $F=\{A \rightarrow B$, $B C \rightarrow D\}$
- Decompose $R$ into $R_{1}=(A, B)$ and $R_{2}=(A, C, D, E)$
- Neither of the dependencies in $F$ contain only attributes from
( $A, C, D, E$ ) so we might be mislead into thinking $R_{2}$ satisfies BCNF.
- In fact, dependency $A C \rightarrow D$ in $F^{+}$shows $R_{2}$ is not in BCNF.


## BCNF Decomposition Algorithm

result : : $\{R\}$;
done := false;
compute $F^{+}$;
while (not done) do
if (there is a schema $R_{i}$ in result that is not in BCNF)
then begin
let $\alpha \rightarrow \beta$ be a nontrivial functional dependency that holds on $R_{i}$ such that $\alpha \rightarrow R_{i}$ is not in $F^{+}$, and $\alpha \cap \beta=\varnothing$; result : : (result $\left.-R_{i}\right) \cup\left(R_{i}-\beta\right) \cup(\alpha, \beta) ;$
end
else done := true;
Note: each $R_{i}$ is in BCNF, and decomposition is lossless-join.

## Example of BCNF Decomposition

- class (course_id, title, dept_name, credits, sec_id, semester, year, building, room_number, capacity, time_slot_id)
- Functional dependencies:
- course_id $\rightarrow$ title, dept_name, credits
- building, room_number $\rightarrow$ capacity
- course_id, sec_id, semester, year $\rightarrow$ building, room_number, time_slot_id


## Example of BCNF Decomposition

- A candidate key \{course_id, sec_id, semester, year\}.
- BCNF Decomposition:
- course_id $\rightarrow$ title, dept_name, credits holds
- but course_id is not a superkey.
- We replace class by:
- course(course_id, title, dept_name, credits)
- class-1 (course_id, sec_id, semester, year, building, room_number, capacity, time_slot_id)


## BCNF Decomposition

- course is in BCNF
- How do we know this?
- building, room_number $\rightarrow$ capacity holds on class-1
- but \{building, room_number\} is not a superkey for class-1.
- We replace class-1 by:
. classroom (building, room_number, capacity)
- section (course_id, sec_id, semester, year, building, room_number, time_slot_id)
. classroom and section are in BCNF.


## BCNF and Dependency Preservation

It is not always possible to get a BCNF decomposition that is dependency preserving

- $R=(J, K, L)$
$F=\{J K \rightarrow L$
$L \rightarrow K\}$
Two candidate keys $=J K$ and $J L$
- $R$ is not in BCNF
- Any decomposition of $R$ will fail to preserve

$$
J K \rightarrow L
$$

This implies that testing for $J K \rightarrow L$ requires a join

## Lecture Outline

- An Example
- Functional Dependencies Review
- BCNF Decomposition
- 3NF Decomposition
- Multivalued Decomposition
- Fourth Normal Form
- Database Design Process
- There are some situations where
- BCNF is not dependency preserving, and
- efficient checking for FD violation on updates is important
. Solution: define a weaker normal form, called Third Normal Form (3NF)
- Allows some redundancy (with resultant problems; we will see examples later)
- But functional dependencies can be checked on individual relations without computing a join.
- There is always a lossless-join, dependency-preserving decomposition into 3NF


## 3NF Example

- Relation dept_advisor:
- dept_advisor (s_ID, i_ID, dept_name) $F=\left\{s \_I D\right.$, dept_name $\rightarrow$ i_ID, i_ID $\rightarrow$ dept_name\}
- Two candidate keys: s_ID, dept_name, and i_ID, s_ID
- $R$ is in 3NF
- s_ID, dept_name $\rightarrow$ i_ID
- S_ID,dept_name is a superkey
- i_ID $\rightarrow$ dept_name
- dept_name is contained in a candidate key


## Redundancy in 3NF

- There is some redundancy in this schema
- Example of problems due to redundancy in 3NF

$$
\begin{aligned}
-R & =(J, K, L) \\
F & =\{J K \rightarrow L, L \rightarrow K\}
\end{aligned}
$$

| $J$ | $L$ | $K$ |
| :---: | :---: | :---: |
| $j_{1}$ | $I_{1}$ | $k_{1}$ |
| $j_{2}$ | $I_{1}$ | $k_{1}$ |
| $j_{3}$ | $I_{1}$ | $k_{1}$ |
| null | $I_{2}$ | $k_{2}$ |

- repetition of information (e.g., the relationship $I_{1}, k_{1}$ )
-     - (i_ID, dept_name)
- need to use null values (e.g., to represent the relationship
$-I_{2}, k_{2}$ where there is no corresponding value for $J$ ).
-     - (i_ID, dept_name) if there is no separate relation mapping instructors to departments


## Testing for 3NF

- Optimization: Need to check only FDs in F, need not check all FDs in $F^{+}$.
. Use attribute closure to check for each dependency $\alpha \rightarrow \beta$, if $\alpha$ is a superkey.
- If $\alpha$ is not a superkey, we have to verify if each attribute in $\beta$ is contained in a candidate key of $R$
- this test is rather more expensive, since it involve finding candidate keys
- testing for 3NF has been shown to be NP-hard
- Interestingly, decomposition into third normal form (described shortly) can be done in polynomial time


## 3NF Decomposition Algorithm

Let $F_{c}$ be a canonical cover for $F$;
$i:=0$;
for each functional dependency $\alpha \rightarrow \beta$ in $F_{c}$ do
if none of the schemas $R_{j}, 1 \leq j \leq i$ contains $\alpha \beta$
then begin

$$
\begin{aligned}
& i:=i+1 ; \\
& R_{i}:=\alpha \beta \\
& \text { end }
\end{aligned}
$$

if none of the schemas $R_{j}, 1 \leq j \leq i$ contains a candidate key for $R$ then begin

$$
i:=i+1
$$

$R_{i}:=$ any candidate key for $R$;
end
/* Optionally, remove redundant relations */ repeat
if any schema $R_{j}$ is contained in another schema $R_{k}$
then $I^{*}$ delete $R_{j}{ }^{*} I$
$R_{i}=R ; ;$

$$
i=i-1 ;
$$

return $\left(R_{1}, R_{2}, \ldots, R_{j}\right)$

## 3NF Decomposition Algorithm

- Above algorithm ensures:
- each relation schema $R_{i}$ is in 3NF
- decomposition is dependency preserving and lossless-join
- For proof of correctness see original slides that accompany book


## 3NF Decomposition: An Example

- Relation schema:
cust_banker_branch = (customer id, employee id, branch_name, type )
- The functional dependencies for this relation schema are:
- customer_id, employee_id $\rightarrow$ branch_name, type
- employee_id $\rightarrow$ branch_name
- customer_id, branch_name $\rightarrow$ employee_id


## 3NF Decomposition: An Example

- We first compute a canonical cover
- branch_name is extraneous in the r.h.s. of the $1^{\text {st }}$ dependency
- No other attribute is extraneous, so we get $F_{C}=$ customer_id, employee_id $\rightarrow$ type employee_id $\rightarrow$ branch_name customer_id, branch_name $\rightarrow$ employee_id


## 3NF Decomposition Example

- The for loop generates following 3NF schema:
(customer_id, employee_id, type )
(employee id, branch_name)
(customer_id, branch_name,
employee_id)
- Observe that (customer_id, employee_id, type ) contains a candidate key of the original schema, so no further relation schema needs be added


## 3NF Decomposition Example

- At end of for loop, detect and delete schemas, such as (employee id, branch_name), which are subsets of other schemas
- result will not depend on the order in which FDs are considered
- The resultant simplified 3NF schema is:
(customer_id, employee_id, type) (customer_id, branch_name, employee_id)


## Comparison of BCNF and 3NF

- It is always possible to decompose a relation into a set of relations that are in 3NF such that:
- the decomposition is lossless
- the dependencies are preserved
- It is always possible to decompose a relation into a set of relations that are in BCNF such that:
- the decomposition is lossless
- it may not be possible to preserve dependencies.


## Design Goals

- Goal for a relational database design is:
- BCNF.
- Lossless join.
- Dependency preservation.
- If we cannot achieve this, we accept one of
- Lack of dependency preservation
- Redundancy due to use of 3NF


## Design Goals

- Interestingly, SQL does not provide a direct way of specifying functional dependencies other than superkeys.
Can specify FDs using assertions, but they are expensive to test, (and currently not supported by any of the widely used databases!)
- Even if we had a dependency preserving decomposition, using SQL we would not be able to efficiently test a functional dependency whose left hand side is not a key.


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- Multivalued Decomposition
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## Multivalued Dependencies

- Suppose we record names of children, and phone numbers for instructors:
- inst_child(ID, child_name)
- inst_phone(ID, phone_number)
- If we were to combine these schemas to get
- inst_info(ID, child_name, phone_number)
- Example data:
(99999, David, 512-555-1234)
(99999, David, 512-555-4321)
(99999, William, 512-555-1234)
(99999, William, 512-555-4321)
- This relation is in BCNF


## Multivalued Dependencies (MVDs)

- Let $R$ be a relation schema and let $\alpha \subseteq R$ and $\beta \subseteq R$. The multivalued dependency

$$
\alpha \rightarrow \rightarrow \beta
$$

holds on $R$ if in any legal relation $r(R)$, for all pairs for tuples $t_{1}$ and $t_{2}$ in $r$ such that $t_{1}[\alpha]=t_{2}[\alpha]$, there exist tuples $t_{3}$ and $t_{4}$ in $r$ such that:

$$
t_{1}[\alpha]=t_{2}[\alpha]=t_{3}[\alpha]=t_{4}[\alpha]
$$

$$
t_{3}[\beta]=t_{1}[\beta]
$$

$$
t_{3}[R-\beta]=t_{2}^{\prime}[R-\beta]
$$

$$
t_{4}[\beta]=t_{2}[\beta]
$$

$$
t_{4}[R-\beta]=t_{1}[R-\beta]
$$

# Multivalued Dependencies (MVDs) <br> - Tabular representation of $\alpha \rightarrow \rightarrow \beta$ 

|  | $\alpha$ | $\beta$ | $R-\alpha-\beta$ |
| :---: | :---: | :---: | :---: |
| $t_{1}$ | $a_{1} \ldots a_{i}$ | $a_{i+1} \ldots a_{j}$ | $a_{j+1} \ldots a_{n}$ |
| $t_{2}$ | $a_{1} \ldots a_{i}$ | $b_{i+1} \ldots b_{j}$ | $b_{j+1} \ldots b_{n}$ |
| $t_{3}$ | $a_{1} \ldots a_{i}$ | $a_{i+1} \ldots a_{j}$ | $b_{j+1} \ldots b_{n}$ |
| $t_{4}$ | $a_{1} \ldots a_{i}$ | $b_{i+1} \ldots b_{j}$ | $a_{j+1} \ldots a_{n}$ |

## Example

- In our example: $I D \rightarrow$ child_name
$I D \rightarrow \rightarrow$ phone_number
- The above formal definition is supposed to formalize the notion that given a particular value of $Y(I D)$ it has associated with it a set of values of $Z$ (child_name) and a set of values of $W$ (phone_number), and these two sets are in some sense independent of each other.


## Use of Multivalued Dependencies

- We use multivalued dependencies in two ways:

1. To test relations to determine whether they are legal under a given set of functional and multivalued dependencies
2. To specify constraints on the set of legal relations. We shall thus concern ourselves only with relations that satisfy a given set of functional and multivalued dependencies.

- If a relation $r$ fails to satisfy a given multivalued dependency, we can construct a relations $r^{\prime}$ that does satisfy the multivalued dependency by adding tuples to $r$.


## Theory of MVDs

- From the definition of multivalued dependency, we can derive the following rule:
- If $\alpha \rightarrow \beta$, then $\alpha \rightarrow \rightarrow \beta$

That is, every functional dependency is also a multivalued dependency

## Theory of MVDs

- The closure $\mathrm{D}^{+}$of $D$ is the set of all functional and multivalued dependencies logically implied by $D$.
- We can compute $\mathrm{D}^{+}$from $D$, using the formal definitions of functional dependencies and multivalued dependencies.
- We can manage with such reasoning for very simple multivalued dependencies, which seem to be most common in practice


## Theory of MVDs

## The functional dependency

$$
\alpha \rightarrow \beta
$$

holds on $R$ if and only if for any legal relations $r(R)$, whenever any two tuples $t_{1}$ and $t_{2}$ of $r$ agree on the attributes $\alpha$, they also agree on the attributes $\beta$. That is,

$$
\breve{t}_{1}[\alpha]=t_{2}[\alpha] \Rightarrow t_{1}[\beta]=t_{2}[\beta]
$$

| 1 | 4 |
| :--- | :--- |
| 1 | 5 |
| 3 | 7 |

- Let $R$ be a relation schema and let $\alpha \subseteq R$ and $\beta \subseteq R$. The multivalued dependency

$$
\alpha \rightarrow \rightarrow \beta
$$

holds on $R$ if in any legal relation $r(R)$, for all pairs for tuples $t_{1}$ and $t_{2}$ in $r$ such that $t_{1}[\alpha]=t_{2}[\alpha]$, there exist tuples $t_{3}$ and $t_{4}$ in $r$ such that:

$$
t_{1}[\alpha]=t_{2}[\alpha]=t_{3}[\alpha]=t_{4}[\alpha]
$$

$$
t_{3}[\beta]=t_{1}[\beta]
$$

$$
t_{3}[R-\beta]=t_{2}[R-\beta]
$$

$$
t_{4}[\beta]=t_{2}[\beta]
$$

$$
t_{4}[R-\beta]=t_{1}[R-\beta]
$$

## Lecture Outline

- An Example
- Review for Midterm
- BCNF Decomposition
- 3NF Decomposition
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## Fourth Normal Form

- A relation schema $R$ is in $4 N F$ with respect to a set $D$ of functional and multivalued dependencies if for all multivalued dependencies in $D^{+}$of the form $\alpha \rightarrow \rightarrow \beta$, where $\alpha \subseteq R$ and $\beta \subseteq R$, at least one of the following hold:
$-\alpha \rightarrow \beta$ is trivial (i.e., $\beta \subseteq \alpha$ or $\alpha \cup \beta=R$ )
- $\alpha$ is a superkey for schema $R$
. If a relation is in $4 N F$ it is in BCNF

Restriction of Multivalued Dependencies

- The restriction of $D$ to $R_{i}$ is the set $D_{i}$ consisting of
- All functional dependencies in $\mathrm{D}^{+}$that include only attributes of $\mathrm{R}_{\mathrm{i}}$
- All multivalued dependencies of the form $\alpha \rightarrow \rightarrow\left(\beta \cap R_{i}\right)$
where $\alpha \subseteq R_{i}$ and $\alpha \rightarrow \rightarrow \beta$ is in $D^{+}$


## 4NF Decomposition Algorithm

result: $=\{R\}$;
done := false;
compute $D^{+}$;
Let $D_{i}$ denote the restriction of $D^{+}$to $R_{i}$
while (not done)
if (there is a schema $\mathbf{R}_{\mathrm{i}}$ in result that is not in 4NF) then begin
let $\alpha \rightarrow \rightarrow$ be a nontrivial multivalued dependency that holds
on $R_{\mathrm{i}}$ such that $\alpha \rightarrow R_{i}$ is not in $D_{\mathrm{i}}$, and $\alpha \cap \beta=\varphi$; result := (result $\left.-R_{i}\right) \cup\left(R_{i}-\beta\right) \cup(\alpha, \beta)$; end
else done:= true;
Note: each $R_{i}$ is in 4 NF , and decomposition is lossless-join

## Example

$$
\begin{gathered}
R=(A, B, C, G, H, I) \\
F=\{A \rightarrow \rightarrow B \\
B \rightarrow \rightarrow H I \\
C G \rightarrow \rightarrow H\}
\end{gathered}
$$

$R$ is not in 4NF since $A \rightarrow B$ and $A$ is not a superkey for $R$
Decomposition

$$
\begin{aligned}
& R_{1}=(A, B) \\
& R_{2}=(A, C, G, H, I)
\end{aligned}
$$

$$
\text { ( } R_{1} \text { is in 4NF) }
$$

( $R_{2}$ is not in 4 NF , decompose into $\mathrm{R}_{3}$
and $R_{4}$ )

$$
\begin{array}{lr}
R_{3}=(C, G, H) & \left(R_{3} \text { is in } 4 \mathrm{NF}\right) \\
R_{4}=(A, C, G, I) & \left(R_{4} \text { is not in } 4 \mathrm{NF}, \text { decompose into } R_{5}\right.
\end{array}
$$

$-A \xrightarrow{6} \rightarrow B$ and $B \rightarrow H I \rightarrow A \rightarrow H I$, (MVD transitivity), and

- and hence $A \rightarrow \rightarrow$ (MVD restriction to $R_{4}$ )
$R_{5}=(A, l)$
( $R_{5}$ is in 4 NF )
$R_{6}^{5}=(\mathrm{A}, \mathrm{C}, \mathrm{G}) \quad\left(\mathrm{R}_{6}^{5}\right.$ is in 4NF)


## Lecture Outline

- An Example
- Functional Dependencies Review
- BCNF Decomposition
- 3NF Decomposition
- Multivalued Decomposition
- Fourth Normal Form
- Database Design Process


## Overall Database Design Process

- We have assumed schema $R$ is given
- $R$ could have been generated when converting $\mathrm{E}-\mathrm{R}$ diagram to a set of tables.
- $R$ could have been a single relation containing all attributes that are of interest (called universal relation).
- Normalization breaks $R$ into smaller relations.
- $R$ could have been the result of some ad hoc design of relations, which we then test/convert to normal form.


## ER Model and Normalization

- When an E-R diagram is carefully designed, identifying all entities correctly, the tables generated from the E-R diagram should not need further normalization.


## ER Model and Normalization

- However, in a real (imperfect) design, there can be functional dependencies from non-key attributes of an entity to other attributes of the entity
- Example: an employee entity with attributes department_name and building, and a functional dependency department_name $\rightarrow$ building
- Good design would have made department an entity


## ER Model and Normalization

- Functional dependencies from non-key attributes of a relationship set possible...but rare
- Most relationships are binary


## Denormalization for Performance

- May want to use non-normalized schema for performance
- For example, displaying prereqs along with course_id, and title requires join of course with prereq


## Denormalization for Performance

- Alternative 1: Use denormalized relation containing attributes of course as well as prereq with all above attributes
- faster lookup
- extra space and extra execution time for updates
- extra coding work for programmer and possibility of error in extra code


## Denormalization for Performance

- Alternative 2: use a materialized view defined as
course $\bowtie$ prereq
- Benefits and drawbacks same as above, except no extra coding work for programmer and avoids possible errors


## Other Design Issues

- Some aspects of database design are not caught by normalization
- Examples of bad database design, to be avoided


## Other Design Issues

Instead of earnings (company_id, year, amount ), use

```
earnings_2004
earnings_2005
earnings_2006, etc.,
all on the schema (company_id, earnings).
```

Above are in BCNF, but make querying across years difficult and needs new table each year

## Other Design Issues

company_year (company_id, earnings_2004, earnings_2005, earnings_2006)

Also in BCNF, but also makes querying across years difficult and requires new attribute each year.

Is an example of a crosstab, where values for one attribute become column names

Used in spreadsheets, and in data analysis tools

