1. Definitions of $\Theta(g(n))$, $O(g(n))$ and $\Omega(g(n))$.

$\Theta(g(n)) = \{\ f(n)$: there exist positive constants c_1, c_2, and n_0 such that $0 \le c_1 g(n) \le f(n) \le c_2 g(n)$ for all $n \ge n_0$ $\}$.

$O(g(n)) = \{\ f(n)$: there exist positive constants c and n_0 such that $0 \le f(n) \le cg(n)$ for all $n \ge n_0$ $\}$.

$o(g(n)) = \{\ f(n)$: for any positive constant c, there exists a positive constant n_0 such that $0 \le f(n) < cg(n)$ for all $n \ge n_0$ $\}$.

$\Omega(g(n)) = \{\ f(n)$: there exist positive constants c and n_0 such that $0 \le cg(n) \le f(n)$ for all $n \ge n_0$ $\}$.

$\omega(g(n)) = \{\ f(n)$: for any positive constant c, there exists a positive constant n_0 such that $0 \le cg(n) < f(n)$ for all $n \ge n_0$ $\}$.

2. Master Theorem.

Let $a \ge 1$ and $b > 1$ be constants, let $f(n)$ be a function, and let $T(n)$ be defined on the nonnegative integers by the recurrence

$$T(n) = aT(n/b) + f(n),$$

where we interpret n/b to mean either $\lfloor n/b \rfloor$ or $\lceil n/b \rceil$. Then $T(n)$ can be bounded asymptotically as follows.

1. If $f(n) = O(n^{\log_b a - \epsilon})$ for some constant $\epsilon > 0$, then $T(n) = \Theta(n^{\log_b a})$.
2. If $f(n) = \Theta(n^{\log_b a})$ then $T(n) = \Theta(n^{\log_b a} \lg n)$.
3. If $f(n) = \Omega(n^{\log_b a + \epsilon})$ for some constant $\epsilon > 0$, and if $af(n/b) \le cf(n)$ for some constant $c < 1$ and all sufficiently large n, then $T(n) = \Theta(f(n))$.

3. Exponential Identities. For all real $a \ne 0$, m and n:

$a^0 = 1$. $\quad a^1 = a$. $\quad a^{-1} = 1/a$. $\quad (a^m)^n = a^{mn}$.

$(a^m)^n = (a^n)^m$. $\quad a^m a^n = a^{m+n}$. $\quad a^m / a^n = a^{m-n}$.

4. Logarithmic Identities. For all $a > 0$, $b > 0$, $c > 0$ and n:

$a = b^{\log_b a}$. $\quad \log_c(ab) = (\log_c a) + (\log_c b)$. $\quad \log_b a^n = n \log_b a$. $\quad \log_b a = \dfrac{\log_c a}{\log_c b}$.

$\log_b(1/a) = -\log_b a$. $\quad \log_b a = 1/\log_a b$. $\quad a^{\log_b n} = n^{\log_b a}$. $\quad n^{1/\log_2 n} = 2$.

5. Some summations.

$$\sum_{k=1}^{n} k = \frac{1}{2} n(n+1). \qquad \sum_{k=0}^{n} x^k = \frac{x^{n+1} - 1}{x - 1}. \text{ for real } x \ne 1$$

$$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}. \text{ when } |x| < 1. \qquad \sum_{k=0}^{\infty} kx^k = \frac{x}{(1-x)^2}. \text{ when } |x| < 1.$$

$$\sum_{k=1}^{n} 1/k = \ln n + O(1). \qquad \sum_{k=1}^{n-1} k \lg k \le \frac{1}{2} n^2 \lg n - \frac{1}{8} n^2 \le n^2 \lg n.$$