CMSC 313
COMPUTER ORGANIZATION
&
ASSEMBLY LANGUAGE PROGRAMMING

LECTURE 22, FALL 2012

TOPICS TODAY

- Circuits for Addition
- Standard Logic Components
- Logisim Demo

CIRCUITS FOR ADDITION

3.5 Combinational Circuits

- Combinational logic circuits give us many useful devices.
- One of the simplest is the *half adder*, which finds the sum of two bits.
- We can gain some insight as to the construction of a half adder by looking at its truth table, shown at the right.

Inputs		Outputs	
X	Y	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Half Adder

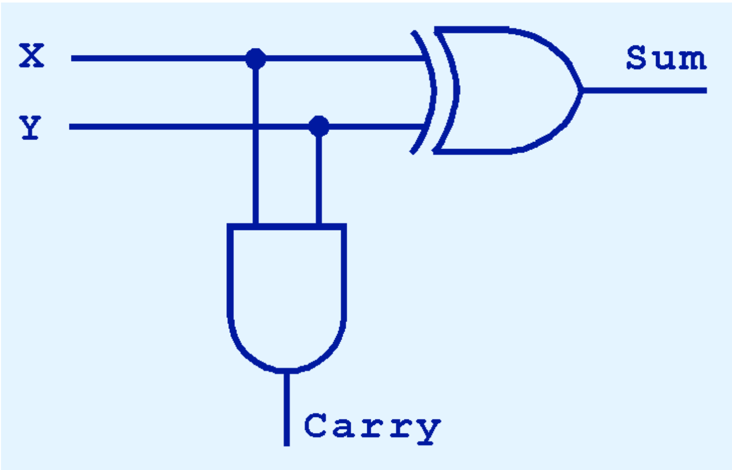
- Inputs: A and B
- Outputs: $S =$ lower bit of $A + B$, $c_{\text{out}} =$ carry bit

A	B	S	c_{out}
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

- Using Sum-of-Products: $S = \overline{A}B + A\overline{B}$, $c_{\text{out}} = AB$.
- Alternatively, we could use XOR: $S = A \oplus B$.

3.5 Combinational Circuits

- As we see, the sum can be found using the XOR operation and the carry using the AND operation.



Inputs		Outputs	
X	Y	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

3.5 Combinational Circuits

- We can change our half adder into to a full adder by including gates for processing the carry bit.
- The truth table for a full adder is shown at the right.

Inputs			Outputs	
X	Y	Carry In	Sum	Carry Out
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Full Adder

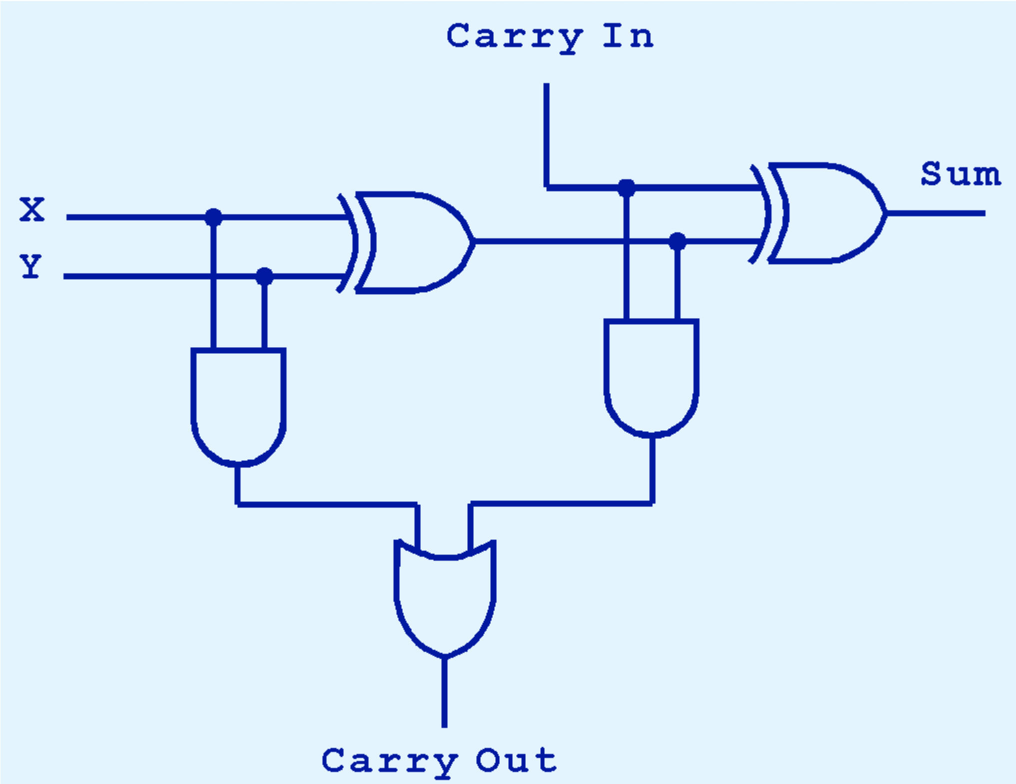
- Inputs: A, B and c_{in}
- Outputs: $S =$ lower bit of $A + B$, $c_{\text{out}} =$ carry bit

A	B	c_{in}	S	c_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

- $S = \overline{A}\,\overline{B}C + \overline{A}B\overline{C} + A\overline{B}\,\overline{C} + ABC = A \oplus B \oplus C.$
- $c_{\text{out}} = \mathrm{MAJ3} = AB + BC + AC.$

3.5 Combinational Circuits

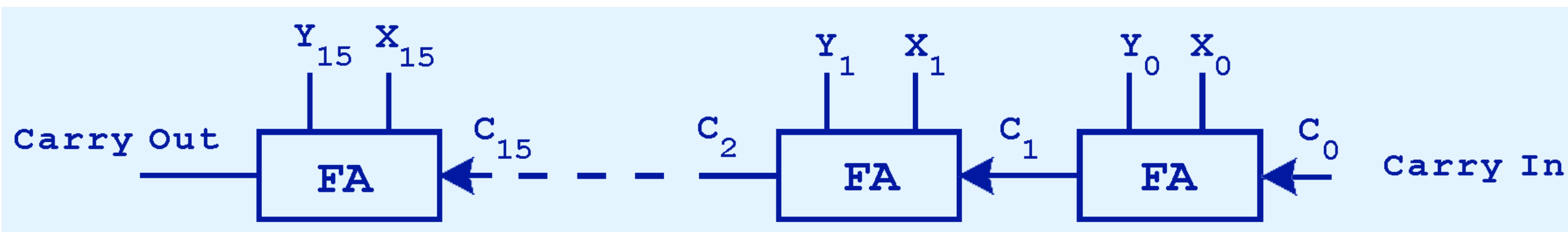
- Here's our completed full adder.



Inputs			Outputs	
X	Y	Carry In	Sum	Carry Out
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

3.5 Combinational Circuits

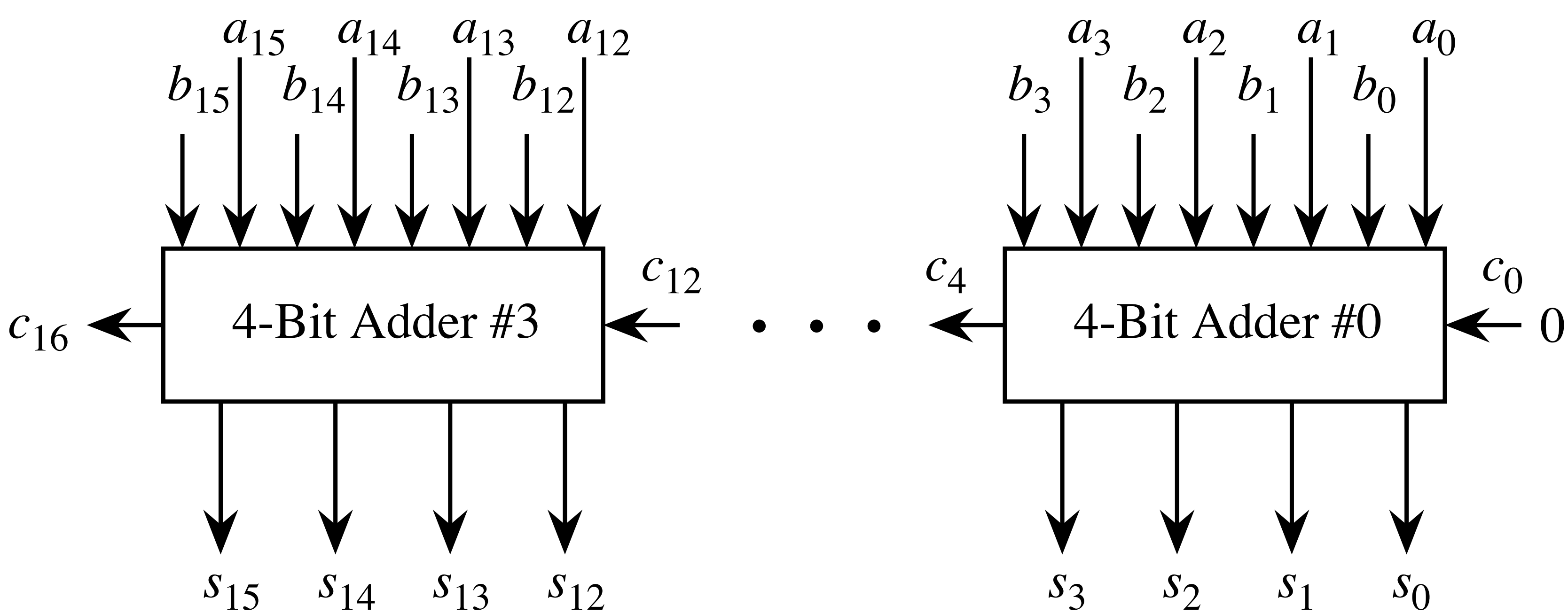
- Just as we combined half adders to make a full adder, full adders can connected in series.
- The carry bit “ripples” from one adder to the next; hence, this configuration is called a *ripple-carry adder*.



Today’s systems employ more efficient adders.

Constructing Larger Adders

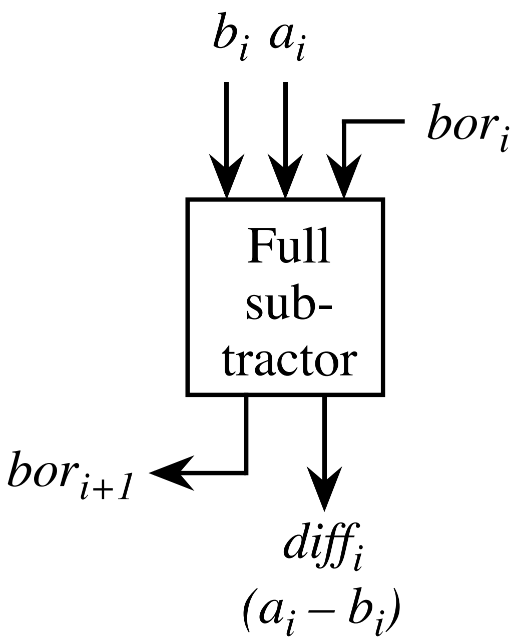
- A 16-bit adder can be made up of a cascade of four 4-bit ripple-carry adders.



Full Subtractor

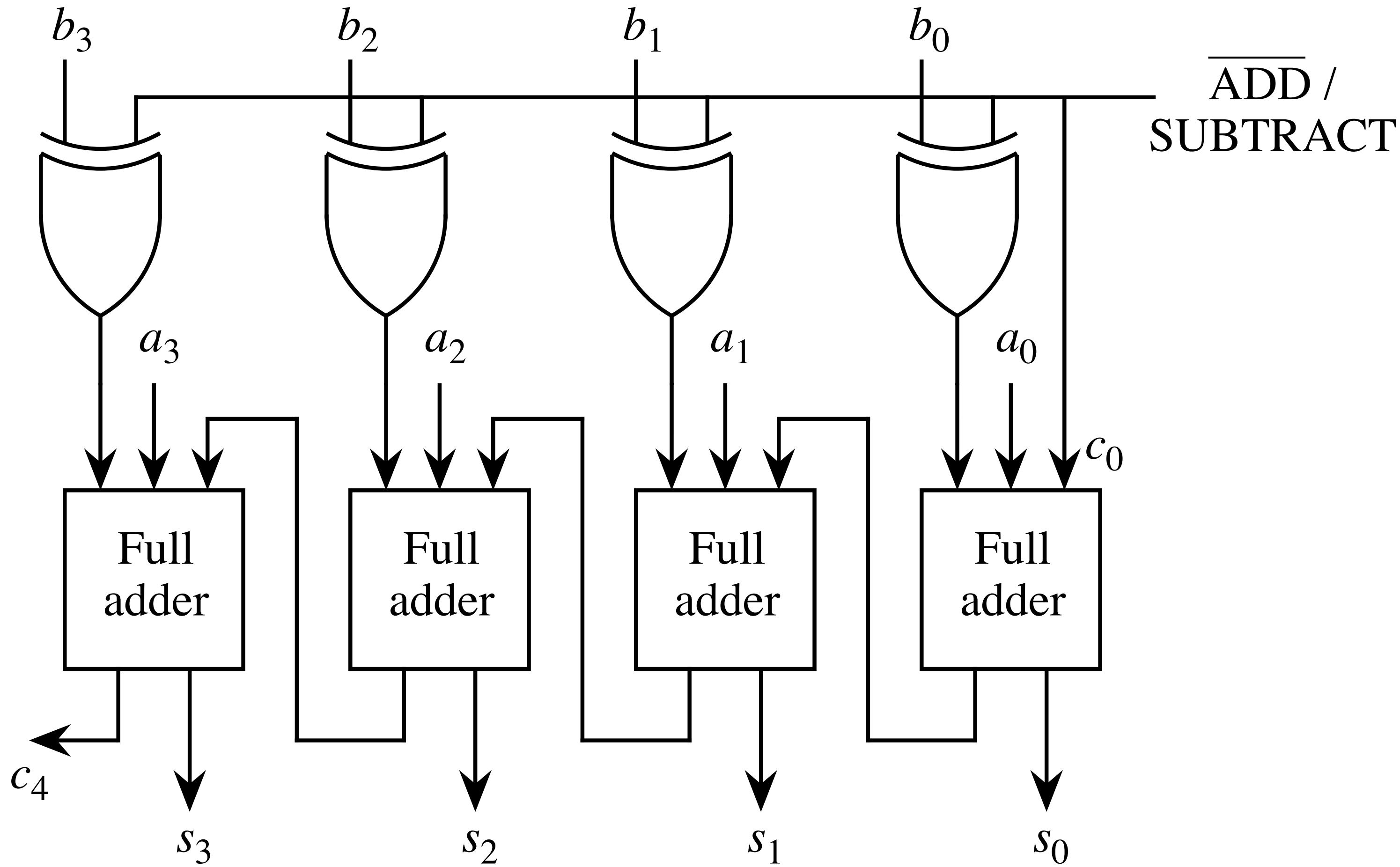
- **Truth table and schematic symbol for a ripple-borrow subtractor:**

a_i	b_i	bor_i	$diff_i$	bor_{i+1}
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1



Combined Adder/Subtractor

- **A single ripple-carry adder can perform both addition and subtraction, by forming the two's complement negative for *B* when subtracting. (Note that +1 is added at c_0 for two's complement.)**



Carry-Lookahead Addition

$$s_i = \overline{a}_i\overline{b}_i c_i + \overline{a}_i b_i \overline{c}_i + a_i \overline{b}_i \overline{c}_i + a_i b_i c_i$$

$$c_{i+1} = b_i c_i + a_i c_i + a_i b_i$$

$$c_{i+1} = a_i b_i + (a_i + b_i) c_i$$

$$c_{i+1} = G_i + P_i c_i$$

- **Carries are represented in terms of G_i (generate) and P_i (propagate) expressions.**

$$G_i = a_i b_i \quad \text{and} \quad P_i = a_i + b_i$$

$$c_0 = 0$$

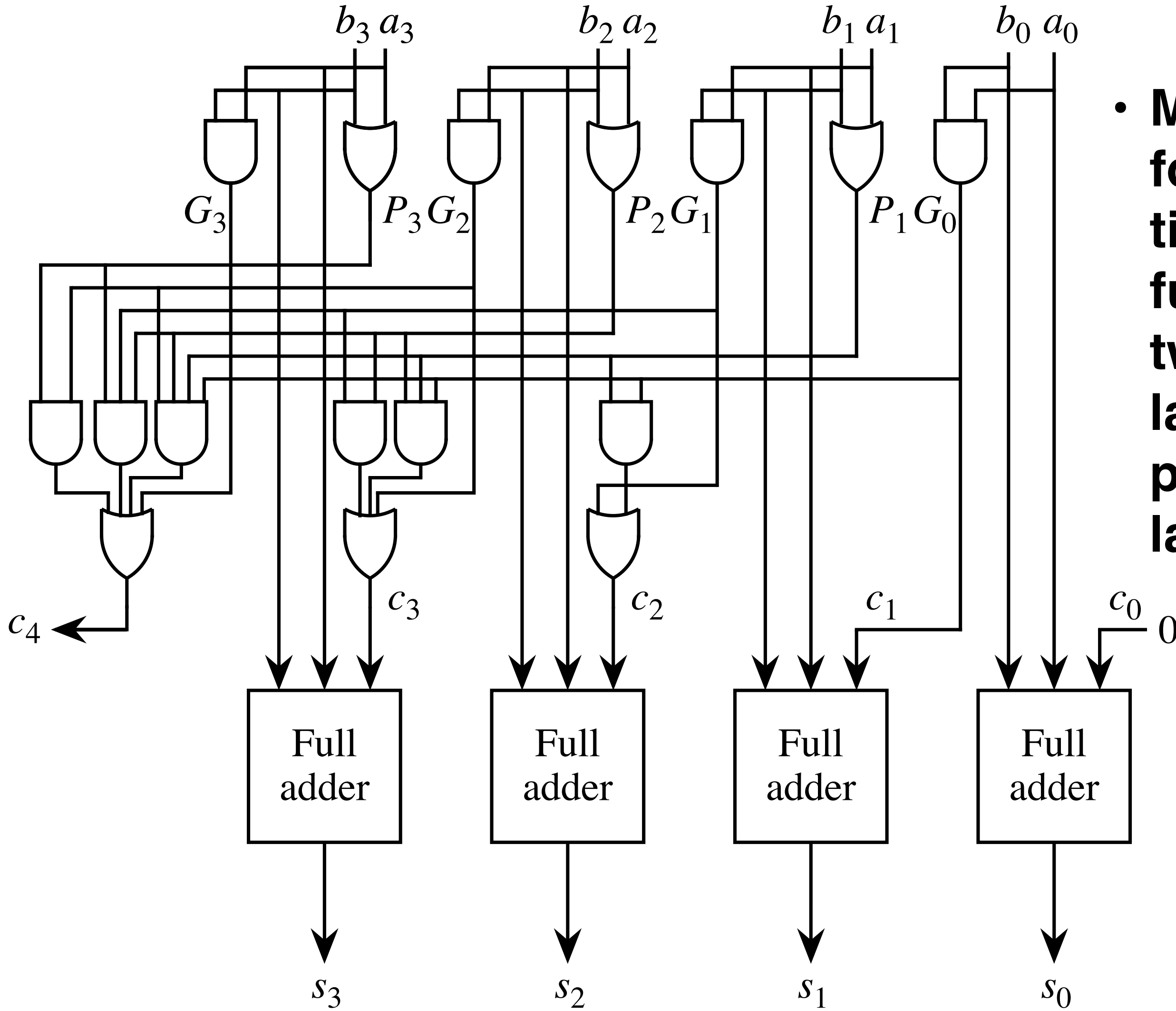
$$c_1 = G_0$$

$$c_2 = G_1 + P_1 G_0$$

$$c_3 = G_2 + P_2 G_1 + P_2 P_1 G_0$$

$$c_4 = G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 G_0$$

Carry Lookahead Adder



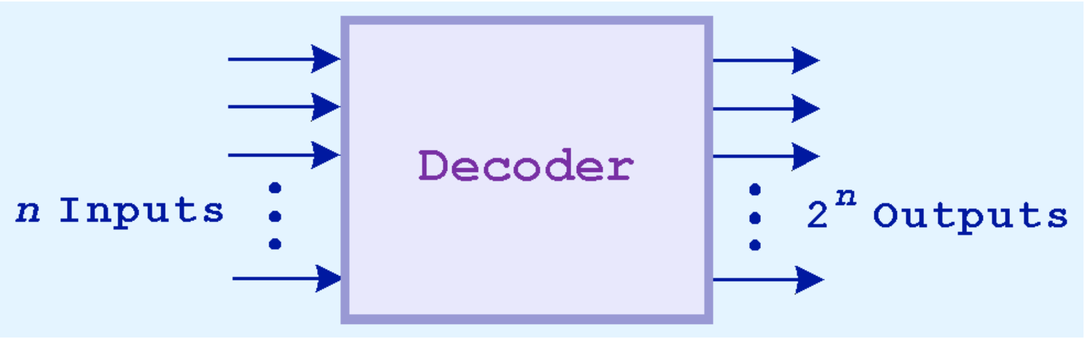
- **Maximum gate delay for the carry generation is only 3. The full adders introduce two more gate delays. Worst case path is 5 gate delays.**

STANDARD LOGIC COMPONENTS

3.5 Combinational Circuits

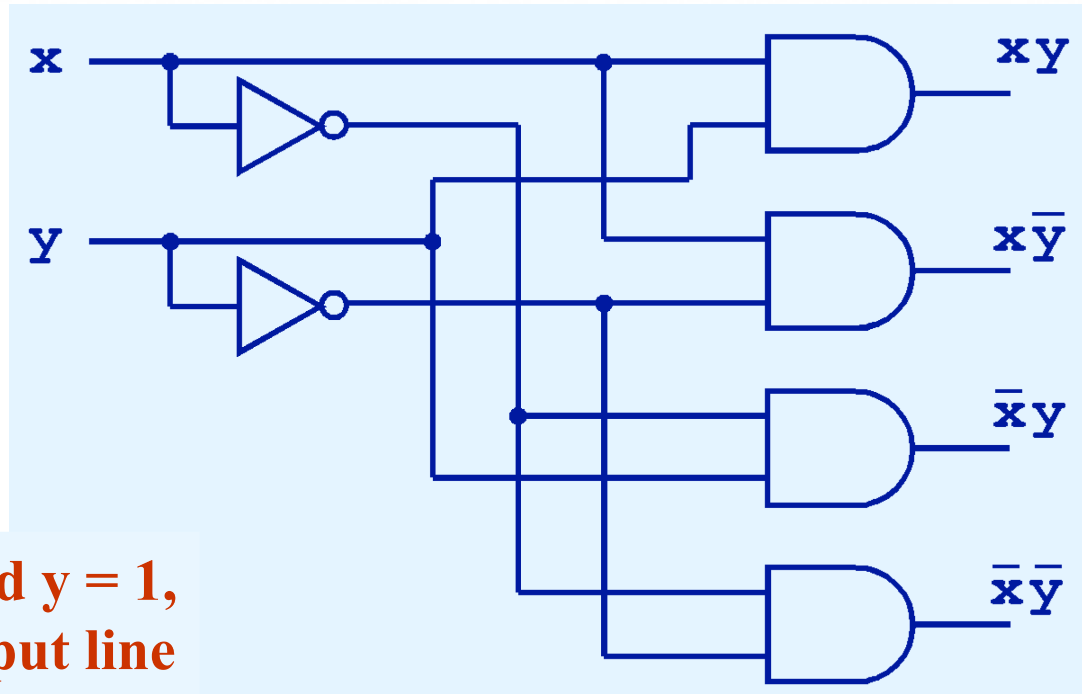
- Decoders are another important type of combinational circuit.
- Among other things, they are useful in selecting a memory location according a binary value placed on the address lines of a memory bus.
- Address decoders with *n* inputs can select any of 2^n locations.

This is a block diagram for a decoder.



3.5 Combinational Circuits

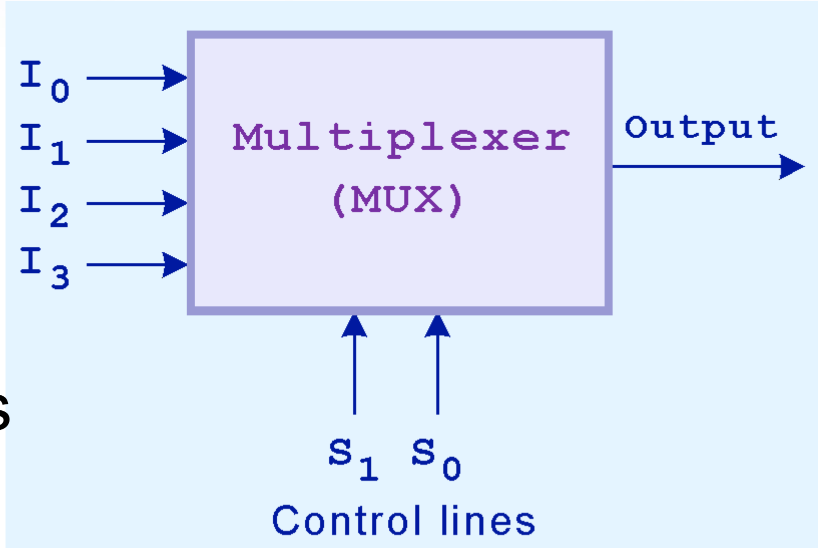
- This is what a 2-to-4 decoder looks like on the inside.



If x = 0 and y = 1, which output line is enabled?

3.5 Combinational Circuits

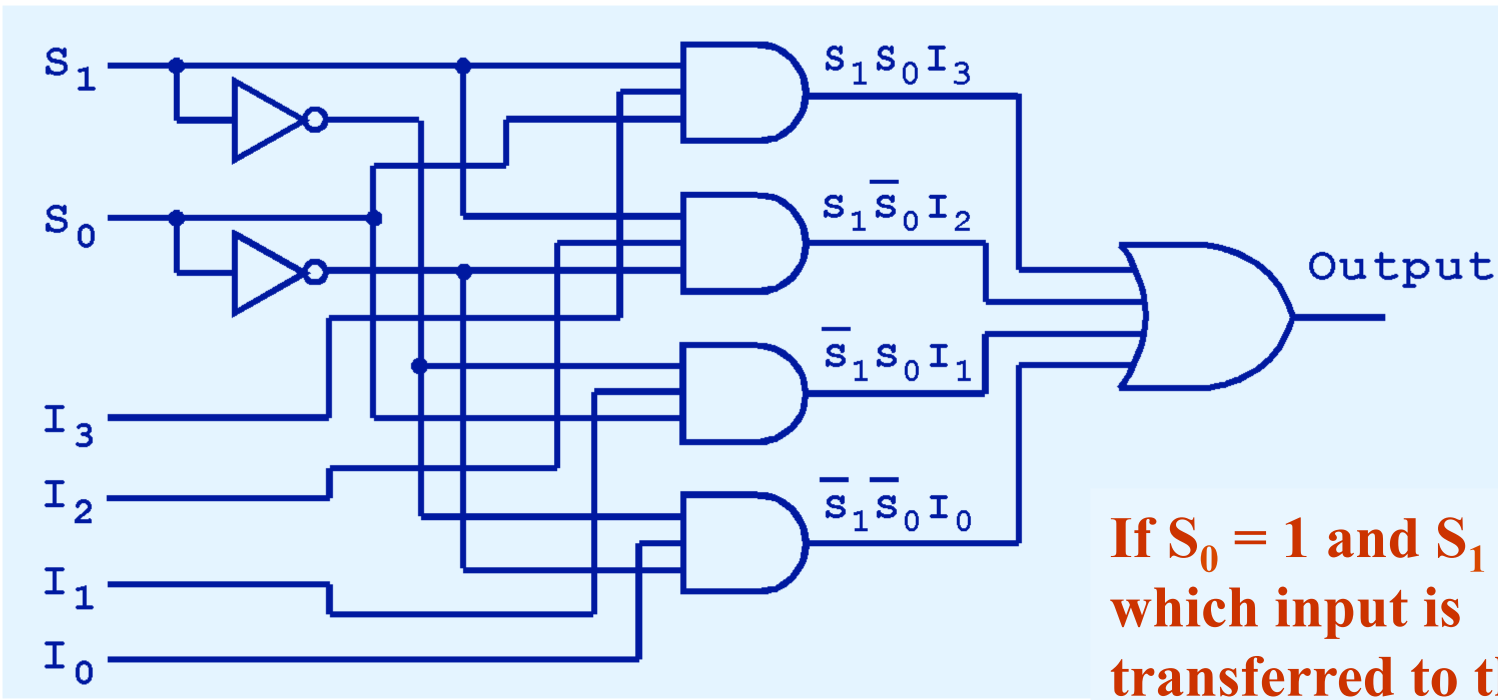
- A multiplexer does just the opposite of a decoder.
- It selects a single output from several inputs.
- The particular input chosen for output is determined by the value of the multiplexer's control lines.
- To be able to select among *n* inputs, $\log_2 n$ control lines are needed.



This is a block diagram for a multiplexer.

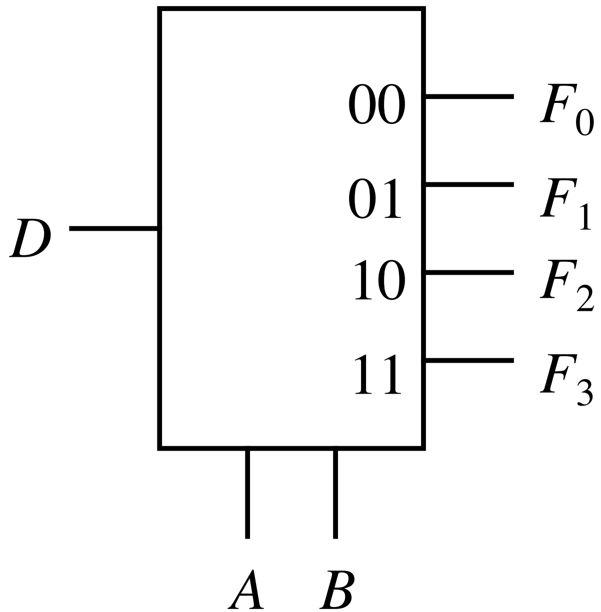
3.5 Combinational Circuits

- This is what a 4-to-1 multiplexer looks like on the inside.



If $S_0 = 1$ and $S_1 = 0$, which input is transferred to the output?

Demultiplexer

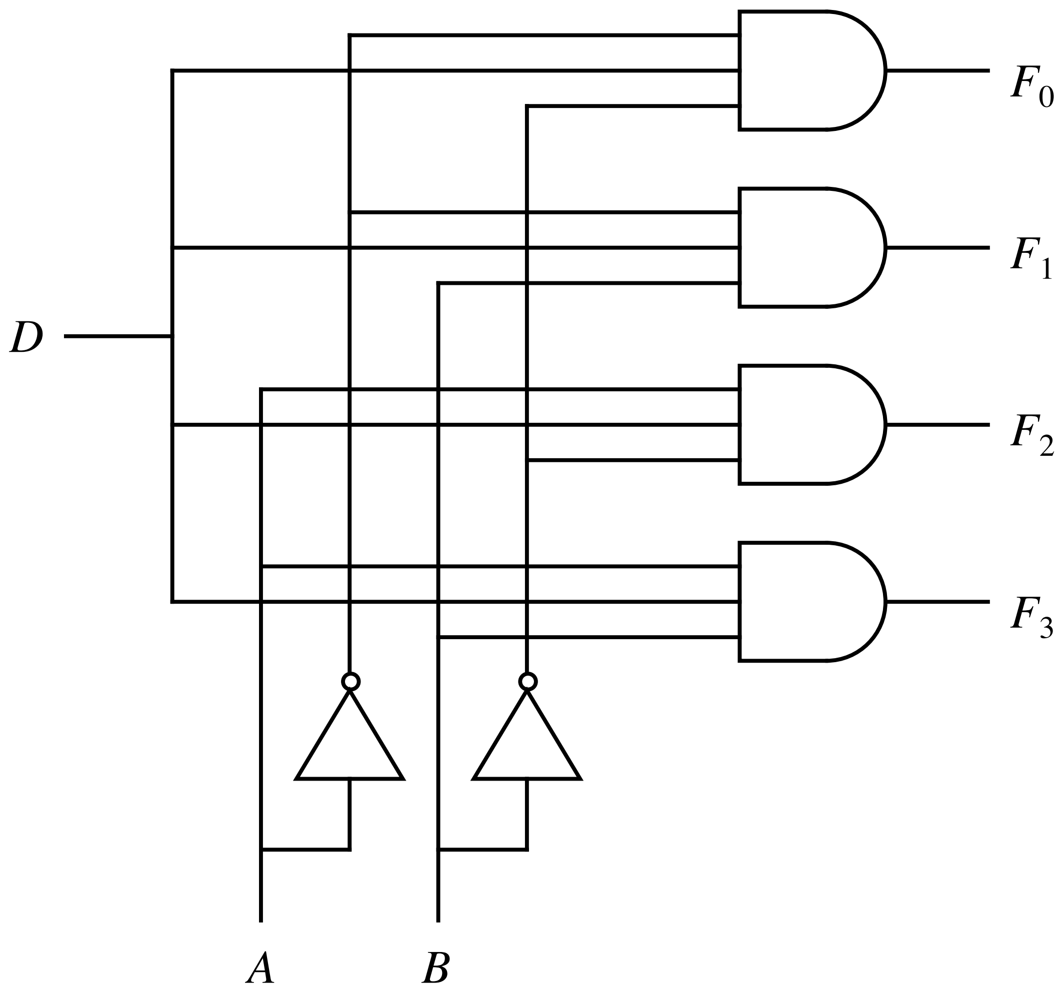


D	A	B	F_0	F_1	F_2	F_3
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	0	0	0
1	0	0	1	0	0	0
1	0	1	0	1	0	0
1	1	0	0	0	1	0
1	1	1	0	0	0	1

$F_0 = D\,\overline{A}\,\overline{B}$ $\qquad F_2 = D\,A\,\overline{B}$

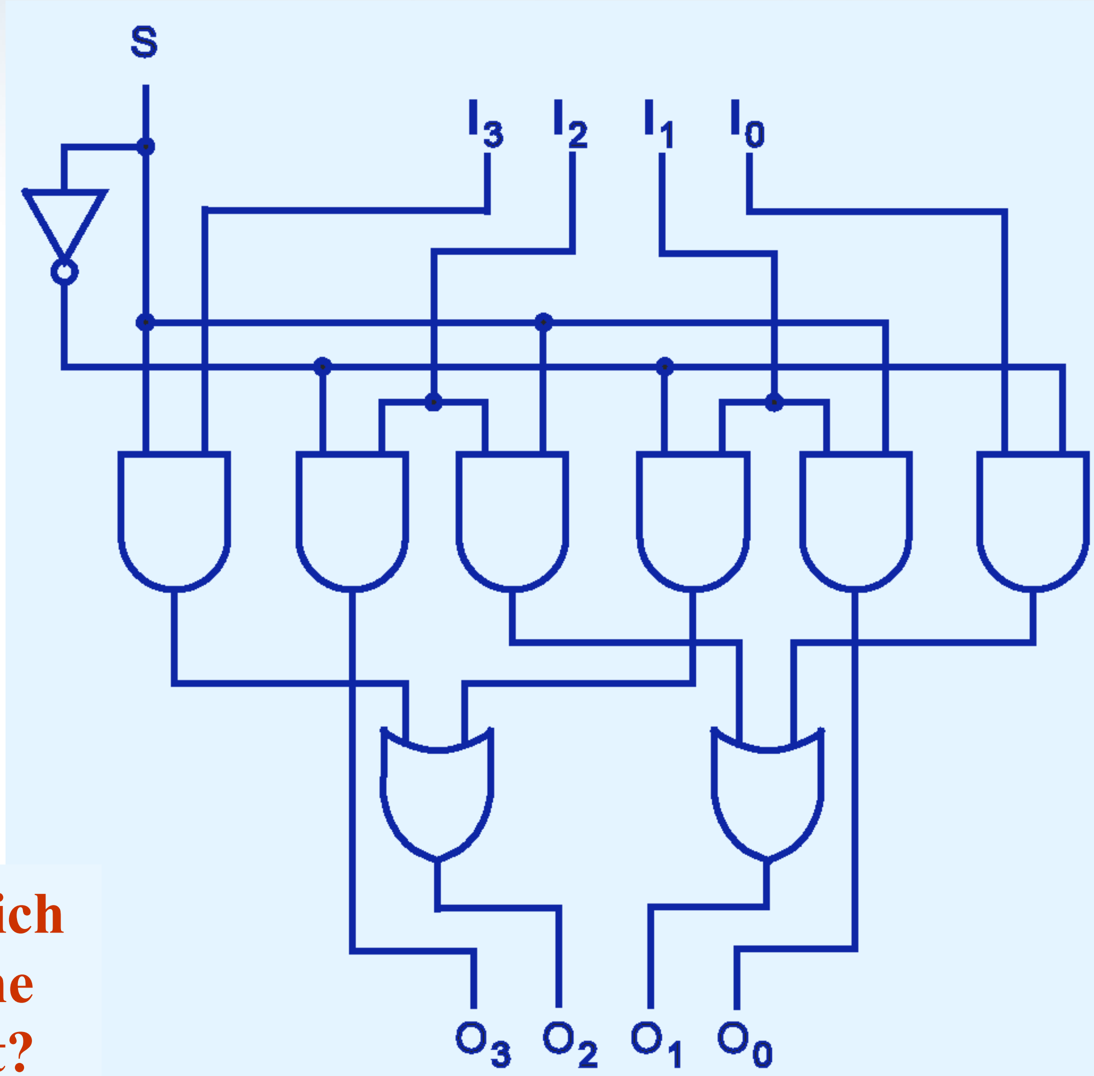
$F_1 = D\,\overline{A}\,B$ $\qquad F_3 = D\,A\,B$

Gate-Level Implementation of DEMUX



3.5 Combinational Circuits

- This shifter moves the bits of a nibble one position to the left or right.



If S = 0, in which direction do the input bits shift?

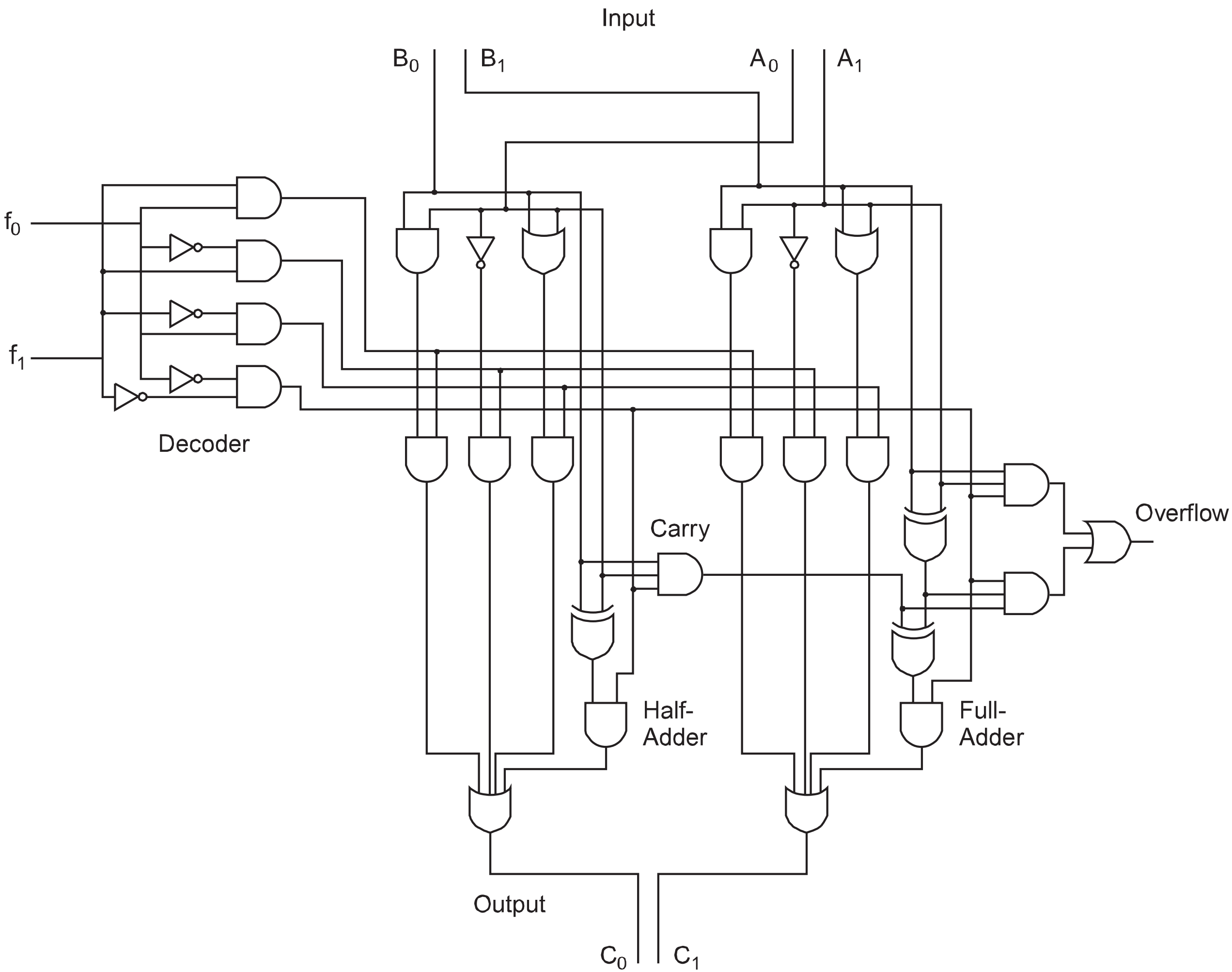


FIGURE 3.17 A Simple Two-Bit ALU

NEXT TIME

- 2-bit ALU
- Flip-flops