Name: ───────────────────────────────

Counting. For each of these questions, *you must show your work and explain your answer. Answers that consist of a single number will not receive very much credit.* When factorials are involved, leave your answer in terms of factorials (e.g., $5!/(3! \cdot 2!)$).

1. You have 9 indistinguishable balls and 3 distinguishable bins. How many different ways are there to place the balls into the bins? Explain your answer.

2. You have 5 indistinguishable red checkers and 10 indistinguishable black checkers. How many ways are there to arrange the checkers so all the red checkers are adjacent to each other? Explain your answer.

3. A student council consists of 9 upperclassmen (juniors and seniors) and 6 underclassmen (sophomores and freshmen). A committee of 7 members must be chosen to serve on a committee. How many groups of 7 can be chosen that contain 4 upperclassmen and 3 underclassmen? Explain your answer.

4. A standard deck of 52 playing cards has 4 suits (spade ♠, heart ♡, diamond ◇ and club ♣) and, in each suit, 13 cards (2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A).

Suppose you are dealt 9 cards. Exactly **two** of the following statements must be true. Which two? For each statement explain why it must be true or how it might be false.

(a) You have at least 3 cards in one of the 4 suits.

(b) You have at least 2 cards in each of the 4 suits.

(c) You have at most 3 cards in one of the 4 suits.

(d) You have at most 4 cards in each of the 4 suits.