Introduction, Data Representation I

CMSC 313
Sections 01, 02

[Review of Syllabus, Web pages]

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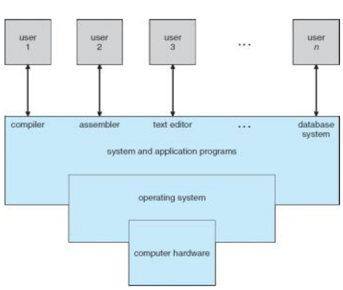
Four Components of a Computer System



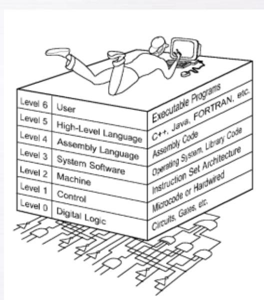
Adapted fm Silberschatz, Galvin & Gagne, 2013

1.6 The Computer Level Hierarchy

- Computers consist of many things besides chips.
- Before a computer can do anything worthwhile, it must also use software.
- Writing complex programs requires a “divide and conquer” approach, where each program module solves a smaller problem.
- Complex computer systems employ a similar technique through a series of virtual machine layers.

1.6 The Computer Level Hierarchy

- Each virtual machine layer is an abstraction of the level below it.
- The machines at each level execute their own particular instructions, calling upon machines at lower levels to perform tasks as required.
- Computer circuits ultimately carry out the work.



1.6 The Computer Level Hierarchy

- Level 6: The User Level
  - Program execution and user interface level.
  - The level with which we are most familiar.
- Level 5: High-Level Language Level
  - The level with which we interact when we write programs in languages such as C, Pascal, Lisp, and Java.

1.6 The Computer Level Hierarchy

- Level 4: Assembly Language Level
  - Acts upon assembly language produced from Level 5, as well as instructions programmed directly at this level.
- Level 3: System Software Level
  - Controls executing processes on the system.
  - Protects system resources.
  - Assembly language instructions often pass through Level 3 without modification.

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1.6 The Computer Level Hierarchy

- Level 2: Machine Level
  - Also known as the Instruction Set Architecture (ISA) Level.
  - Consists of instructions that are particular to the architecture of the machine.
  - Programs written in machine language need no compilers, interpreters, or assemblers.

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1.6 The Computer Level Hierarchy

- Level 1: Control Level
  - A *control unit* decodes and executes instructions and moves data through the system.
  - Control units can be *microprogrammed* or *hardwired*.
  - A microprogram is a program written in a low-level language that is implemented by the hardware.
  - Hardwired control units consist of hardware that directly executes machine instructions.

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1.6 The Computer Level Hierarchy

- Level 0: Digital Logic Level
  - This level is where we find digital circuits (the chips).
  - Digital circuits consist of gates and wires.
  - These components implement the mathematical logic of all other levels.

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1.8 The von Neumann Model

- On the ENIAC, all programming was done at the digital logic level.
- Programming the computer involved moving plugs and wires.
- A different hardware configuration was needed to solve every unique problem type.

Configuring the ENIAC to solve a "simple" problem required many days labor by skilled technicians.

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1.8 The von Neumann Model

- Inventors of the ENIAC, John Mauchley and J. Presper Eckert, conceived of a computer that could store instructions in memory.
- The invention of this idea has since been ascribed to a mathematician, John von Neumann, who was a contemporary of Mauchley and Eckert.
- Stored-program computers have become known as von Neumann Architecture systems.

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1.8 The von Neumann Model

- Today's stored-program computers have the following characteristics:
  - Three hardware systems:
    - A central processing unit (CPU)
    - A main memory system
    - An I/O system
  - The capacity to carry out sequential instruction processing.
  - A single data path between the CPU and main memory.
    - This single path is known as the *von Neumann bottleneck*.

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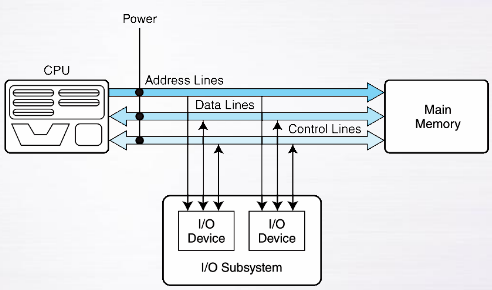
4.3 The Bus



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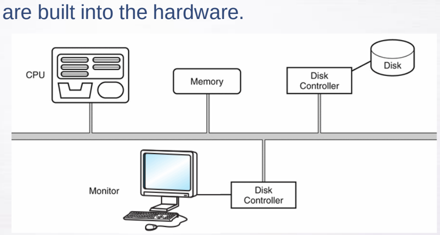
4.3 The Bus

- A multipoint bus is shown below.
- Because a multipoint bus is a shared resource, access to it is controlled through protocols, which are built into the hardware.



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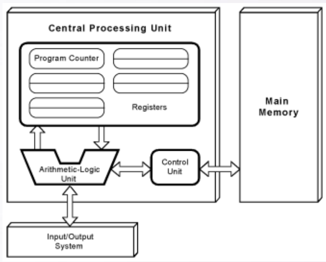
1.8 The von Neumann Model

- This is a general depiction of a von Neumann system:
- These computers employ a fetch-decode-execute cycle to run programs as follows . . .



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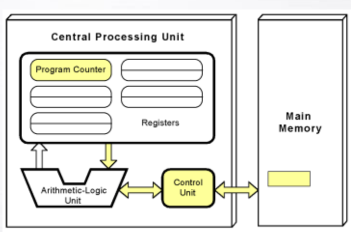
1.8 The von Neumann Model

- **The control unit fetches the next instruction from memory using the program counter to determine where the instruction is located.**



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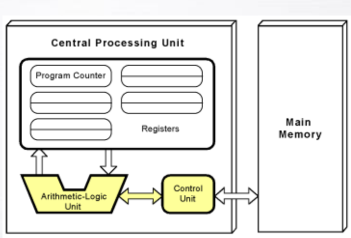
1.8 The von Neumann Model

- **The instruction is decoded into a language that the ALU can understand.**



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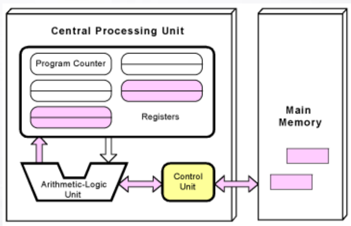
1.8 The von Neumann Model

- Any data operands required to execute the instruction are fetched from memory and placed into registers within the CPU.



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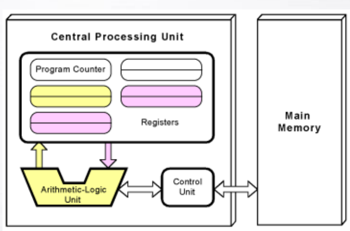
1.8 The von Neumann Model

- The ALU executes the instruction and places results in registers or memory.



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BASE CONVERSION

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2.1 Introduction

- A *bit* is the most basic unit of information in a computer.
  - It is a state of "on" or "off" in a digital circuit.
  - Sometimes these states are "high" or "low" voltage instead of "on" or "off.."
- A *byte* is a group of eight bits.
  - A byte is the smallest possible *addressable* unit of computer storage.
  - The term, "addressable," means that a particular byte can be retrieved according to its location in memory.

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2.1 Introduction

- A *word* is a contiguous group of bytes.
  - Words can be any number of bits or bytes.
  - Word sizes of 16, 32, or 64 bits are most common.
  - In a word-addressable system, a word is the smallest addressable unit of storage.
- A group of four bits is called a *nibble*.
  - Bytes, therefore, consist of two nibbles: a "high-order nibble," and a "low-order" nibble.

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2.2 Positional Numbering Systems

- Bytes store numbers using the position of each bit to represent a power of 2.
  - The binary system is also called the base-2 system.
  - Our decimal system is the base-10 system. It uses powers of 10 for each position in a number.
  - Any integer quantity can be represented exactly using any base (or *radix*).

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2.2 Positional Numbering Systems

- The decimal number 947 in powers of 10 is:

$$9 \times 10^2 + 4 \times 10^1 + 7 \times 10^0$$

- The decimal number 5836.47 in powers of 10 is:

$$5 \times 10^3 + 8 \times 10^2 + 3 \times 10^1 + 6 \times 10^0 + 4 \times 10^{-1} + 7 \times 10^{-2}$$

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2.2 Positional Numbering Systems

- The binary number 11001 in powers of 2 is:

$$1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$
$$= 16 + 8 + 0 + 0 + 1 = 25$$

- When the radix of a number is something other than 10, the base is denoted by a subscript.
  - Sometimes, the subscript 10 is added for emphasis:

$$11001_2 = 25_{10}$$

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2.3 Converting Between Bases

- Because binary numbers are the basis for all data representation in digital computer systems, it is important that you become proficient with this radix system.
- Your knowledge of the binary numbering system will enable you to understand the operation of all computer components as well as the design of instruction set architectures.

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2.3 Converting Between Bases

- In an earlier slide, we said that every integer value can be represented exactly using any radix system.
- There are two methods for radix conversion: the subtraction method and the division remainder method.
- The subtraction method is more intuitive, but cumbersome. It does, however reinforce the ideas behind radix mathematics.

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2.3 Converting Between Bases

- **Suppose we want to convert the decimal number 190 to base 3.**
  - We know that $3^5 = 243$ so our result will be less than six digits wide. The largest power of 3 that we need is therefore $3^4 = 81$, and $81 \times 2 = 162$.
  - Write down the 2 and subtract 162 from 190, giving 28.

$$\begin{array}{r l} 190 & \\ -\ 162 & = 3^4 \times 2 \\ \hline 28 & \end{array}$$

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2.3 Converting Between Bases

- **Converting 190 to base 3...**
  - The next power of 3 is $3^3 = 27$. We'll need one of these, so we subtract 27 and write down the numeral 1 in our result.
  - The next power of 3, $3^2 = 9$, is too large, but we have to assign a placeholder of zero and carry down the 1.

$$\begin{array}{r l} 190 & \\ -\ 162 & = 3^4 \times 2 \\ \hline 28 & \\ -\ 27 & = 3^3 \times 1 \\ \hline 1 & \\ -\ 0 & = 3^2 \times 0 \\ \hline 1 & \end{array}$$

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2.3 Converting Between Bases

- **Converting 190 to base 3...**
  - $3^1 = 3$ is again too large, so we assign a zero placeholder.
  - The last power of 3, $3^0 = 1$, is our last choice, and it gives us a difference of zero.
  - Our result, reading from top to bottom is:

    $190_{10} = 21001_3$

$$
\begin{array}{rl}
 & 190 \\
- & 162 = 3^4 \times 2 \\
 & 28 \\
- & 27 = 3^3 \times 1 \\
 & 1 \\
- & 0 = 3^2 \times 0 \\
 & 1 \\
- & 0 = 3^1 \times 0 \\
 & 1 \\
- & 1 = 3^0 \times 1 \\
 & 0
\end{array}
$$

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2.3 Converting Between Bases

- Another method of converting integers from decimal to some other radix uses division.
- This method is mechanical and easy.
- It employs the idea that successive division by a base is equivalent to successive subtraction by powers of the base.
- Let's use the division remainder method to again convert 190 in decimal to base 3.

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2.3 Converting Between Bases

- **Converting 190 to base 3...**
  - First we take the number that we wish to convert and divide it by the radix in which we want to express our result.
  - In this case, 3 divides 190 63 times, with a remainder of 1.
  - Record the quotient and the remainder.

$$
\begin{array}{r|rl}
3 & 190 & 1 \\
\hline
 & 63 &
\end{array}
$$

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2.3 Converting Between Bases

- **Converting 190 to base 3...**
  - 63 is evenly divisible by 3.
  - Our remainder is zero, and the quotient is 21.

3	190	1
3	63	0
	21	

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2.3 Converting Between Bases

- **Converting 190 to base 3...**
  - Continue in this way until the quotient is zero.
  - In the final calculation, we note that 3 divides 2 zero times with a remainder of 2.
  - Our result, reading from bottom to top is:

    $190_{10} = 21001_3$

3	190	1
3	63	0
3	21	0
3	7	1
3	2	2
	0	

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2.3 Converting Between Bases

- The binary numbering system is the most important radix system for digital computers.
- However, it is difficult to read long strings of binary numbers -- and even a modestly-sized decimal number becomes a very long binary number.
  - For example: $11010100011011_2 = 13595_{10}$
- For compactness and ease of reading, binary values are usually expressed using the hexadecimal, or base-16, numbering system.

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2.3 Converting Between Bases

- The hexadecimal numbering system uses the numerals 0 through 9 and the letters A through F.
  - The decimal number 12 is C_{16}.
  - The decimal number 26 is $1A_{16}$.
- It is easy to convert between base 16 and base 2, because $16 = 2^4$.
- Thus, to convert from binary to hexadecimal, all we need to do is group the binary digits into groups of four.

A group of four binary digits is called a hextet

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2.3 Converting Between Bases

- Using groups of hextets, the binary number 11010100011011_2 (= 13595_{10}) in hexadecimal is:

0011	0101	0001	1011
3	5	1	B

If the number of bits is not a multiple of 4, pad on the left with zeros.

- Octal (base 8) values are derived from binary by using groups of three bits ($8 = 2^3$):

011	010	100	011	011
3	2	4	3	3

Octal was very useful when computers used six-bit words.

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2.3 Converting Between Bases

- Fractional values can be approximated in all base systems.
- Unlike integer values, fractions do not necessarily have exact representations under all radices.
- The quantity ½ is exactly representable in the binary and decimal systems, but is not in the ternary (base 3) numbering system.

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2.3 Converting Between Bases

- Fractional decimal values have nonzero digits to the right of the decimal point.
- Fractional values of other radix systems have nonzero digits to the right of the *radix point*.
- Numerals to the right of a radix point represent negative powers of the radix:

$$0.47_{10} = 4 \times 10^{-1} + 7 \times 10^{-2}$$
$$0.11_2 = 1 \times 2^{-1} + 1 \times 2^{-2}$$
$$= \tfrac{1}{2} + \tfrac{1}{4}$$
$$= 0.5 + 0.25 = 0.75$$

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2.3 Converting Between Bases

- As with whole-number conversions, you can use either of two methods: a subtraction method or an easy multiplication method.
- The subtraction method for fractions is identical to the subtraction method for whole numbers. Instead of subtracting positive powers of the target radix, we subtract negative powers of the radix.
- We always start with the largest value first, n^{-1}, where n is our radix, and work our way along using larger negative exponents.

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2.3 Converting Between Bases

- **The calculation to the right is an example of using the subtraction method to convert the decimal 0.8125 to binary.**
  - Our result, reading from top to bottom is:

    $0.8125_{10} = 0.1101_2$
  - Of course, this method works with any base, not just binary.

```
  0.8125
- 0.5000  = 2^-1 × 1
  0.3125
- 0.2500  = 2^-2 × 1
  0.0625
-      0  = 2^-3 × 0
  0.0625
- 0.0625  = 2^-4 × 1
       0
```

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2.3 Converting Between Bases

- **Using the multiplication method to convert the decimal 0.8125 to binary, we multiply by the radix 2.**
  - The first product carries into the units place.

```
  .8125
  ×   2
 1.6250
```


2.3 Converting Between Bases

- **Converting 0.8125 to binary . . .**
  - Ignoring the value in the units place at each step, continue multiplying each fractional part by the radix.

```
  .8125
  ×   2
 1.6250

  .6250
  ×   2
 1.2500

  .2500
  ×   2
 0.5000
```


2.3 Converting Between Bases

- **Converting 0.8125 to binary . . .**
  - You are finished when the product is zero, or until you have reached the desired number of binary places.
  - Our result, reading from top to bottom is:

    $0.8125_{10} = 0.1101_2$
  - This method also works with any base. Just use the target radix as the multiplier.

```
  .8125
  ×   2
 1.6250

  .6250
  ×   2
 1.2500

  .2500
  ×   2
 0.5000

  .5000
  ×   2
 1.0000
```

Convert Base 6 to Base 10

123.45_6 = ???.?? $_{10}$
$123_6 = 1*6^2{}_{10}$ [$1*36_{10}$] +
$2*6^1{}_{10}$ [$2*6_{10}$] +
$3*6^0{}_{10}$ [$3*1_{10}$] =
51_{10}
$0.45_6 = 4*6^{-1}{}_{10}$ [$4*1/6_{10}$] +
$5*6^{-2}{}_{10}$ [$5*1/36_{10}$] =
$.80555\ldots_{10}$
$123.45_6 = 51.80555\ldots_{10}$

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Convert Base 10 to Base 6

754.94_{10} = 3254.5 35012 35012 35012...$_6$
754 / 6 = 125 remainder 4
125 / 6 = 20 remainder 5
20 / 6 = 3 remainder 2
3 / 6 = 0 remainder 3

$3254_6 = 3 \times 216_{10} + 2 \times 36_{10} + 5 \times 6_{10} + 4 \times 1_{10}$
$= 754_{10}$

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Convert Base 10 to Base 6

$.94_{10}$ = ???.???$_6$
0.94 x 6 = 5.64 --> 5
0.64 x 6 = 3.84 --> 3
0.84 x 6 = 5.04 --> 5
0.04 x 6 = 0.24 --> 0
0.24 x 6 = 1.44 --> 1
0.44 x 6 = 2.64 --> 2
0.64 x 6 = 3.84 --> 3

0.94_{10} = 0.5 35012 35012 35012...$_6$
$5/6 + 3/36 + 5/216 + 0 + 1/6^5 + 2/6^6 = 0.939986282\ldots_{10}$

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