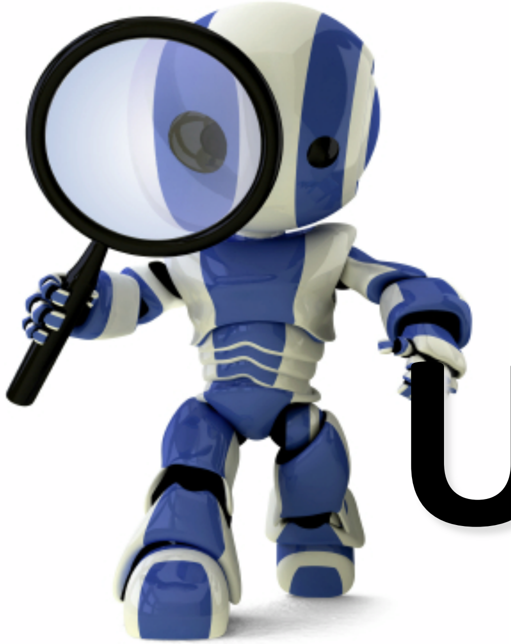


Uninformed Search

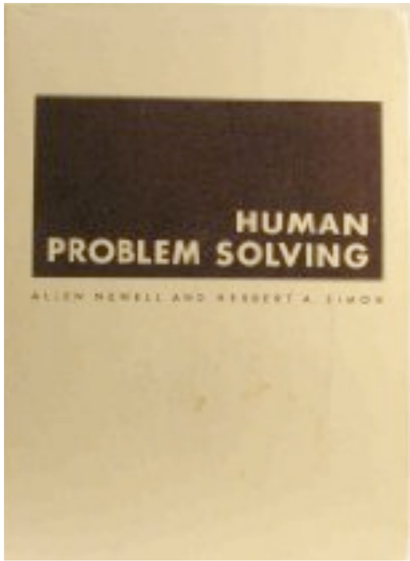
Chapter 3

Some material adopted from notes by Charles R. Dyer, University of Wisconsin-Madison

Today's topics

- Goal-based agents
- Representing states and actions
- Example problems
- Generic state-space search algorithm
- Specific algorithms
  - Breadth-first search
  - Depth-first search
  - Uniform cost search
  - Depth-first iterative deepening
- Example problems revisited

Big Idea



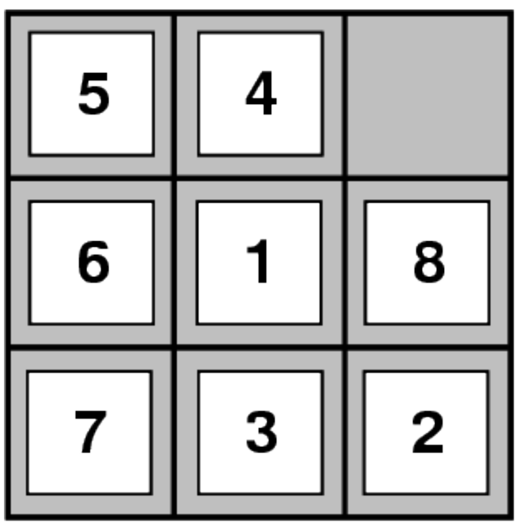
Allen Newell and Herb Simon developed the *problem space principle* as an AI approach in the late 60s/early 70s

"The rational activity in which people engage to solve a problem can be described in terms of (1) a set of **states** of knowledge, (2) **operators** for changing one state into another, (3) **constraints** on applying operators and (4) **control** knowledge for deciding which operator to apply next."

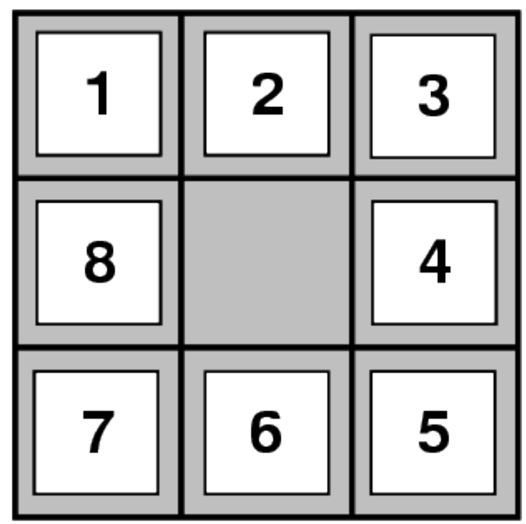
Newell A & *Simon* H A. *Human problem solving*. Englewood Cliffs, NJ: Prentice-Hall. 1972.

Example: 8-Puzzle

Given an initial configuration of 8 numbered tiles on a 3x3 board, move the tiles to produce a desired goal configuration



Start State



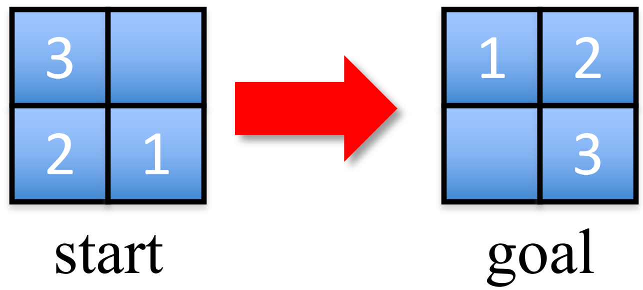
Goal State

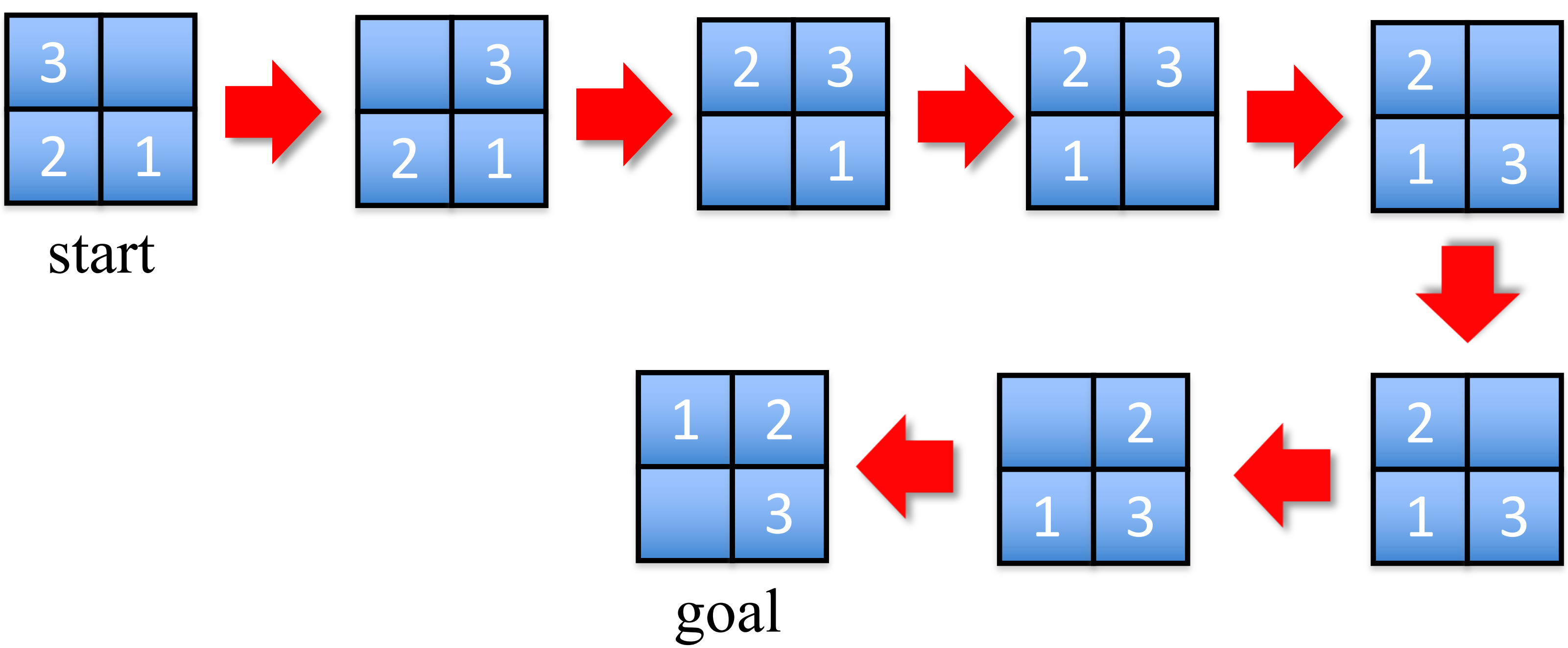
15 puzzle

- Popularized, but not invented, by Sam Loyd
- He offered $1000 to all who could solve it in 1896
- He sold many puzzles
- Its states form two *disjoint* spaces
- There was no path to solution from initial state!

Sam Loyd's 1914 illustration of the unsolvable variation

Simpler: 3-Puzzle

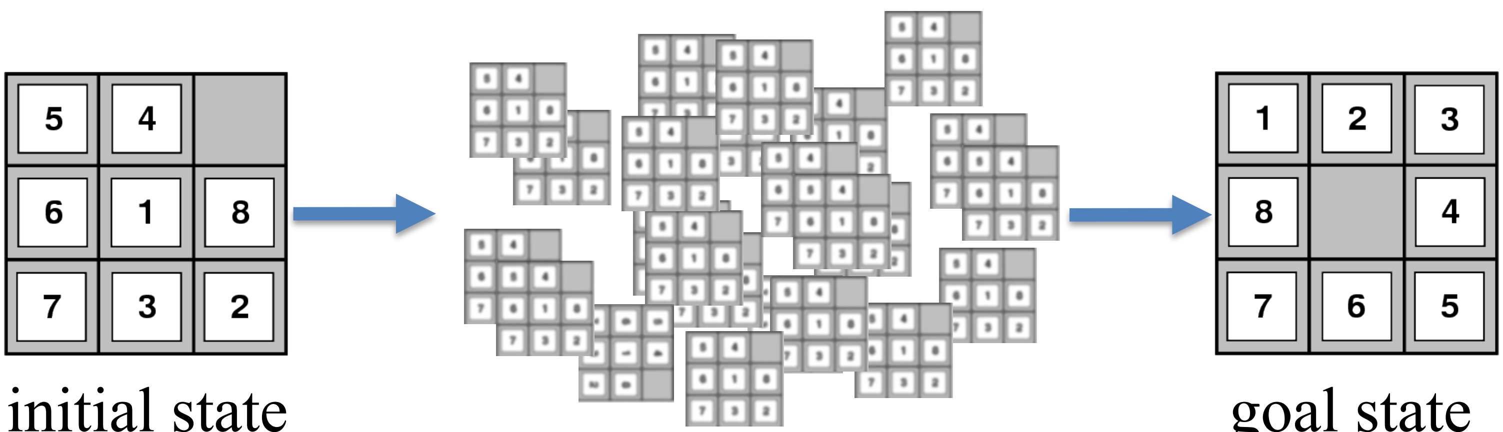




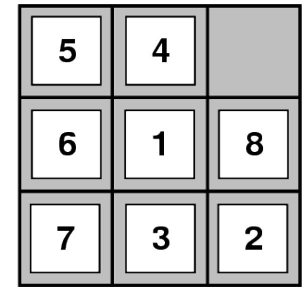
Building goal-based agents

We must answer the following questions

- How do we represent the **state** of the "world"?
- What is the **goal** and how can we recognize it?
- What are the possible **actions**?
- What *relevant* information do we encoded to describe states, actions and their effects and thereby solve the problem?

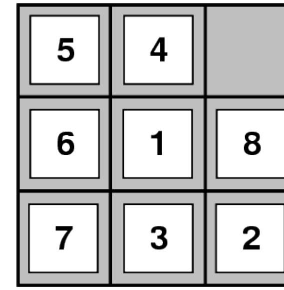


Representing states



- State of an 8-puzzle?

Representing states



- State of an 8-puzzle?
- A 3x3 array of integer in {0..8}
- No integer appears twice
- 0 represents the empty space

- In Python, we might implement this using a nine-character string: "`540681732`"
- And write functions to male the 2D coordinates to an index

What’s the goal to be achieved?

- Describe situation we want to achieve, a set of properties that we want to hold, etc.
- Defining a **goal test** function that when applied to a state returns True or False
- For our problem:

```
def isGoal(state):
    return state == “123405678”
```

What are the actions?

- **Primitive actions** for changing the state

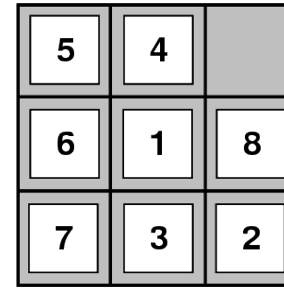
  In a **deterministic** world: no uncertainty in an action’s effects (simple model)

- Given action and description of **current world state**, action completely specifies
  - Whether action ***can*** be applied to the current world (i.e., is it applicable and legal?) and
  - What state ***results*** after action is performed in the current world (i.e., no need for *history* information to compute the next state)

Representing actions

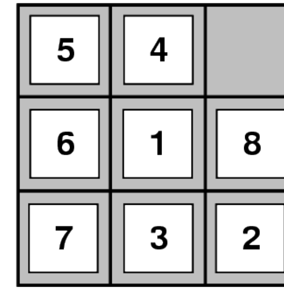
- Actions ideally considered as **discrete events** that occur at an **instant of time**
- Example, in a planning context
  - If state:inClass and perform action:goHome, then next state is state:atHome
  - There's no time where you're neither in class nor at home (i.e., in the state of "going home")

Representing actions



- Actions for 8-puzzle?

Representing actions



- Actions for 8-puzzle?

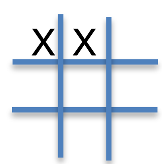
- Number of actions/operators depends on the **representation** used in describing a state
  - Specify 4 possible moves for each of the 8 tiles, resulting in a total of **4*8=32 actions**
  - Or, Specify four moves for “blank” square and we only need **4 actions**

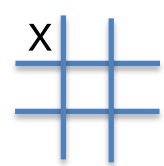
- **Representational shift can simplify a problem!**

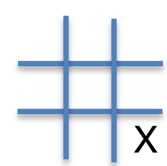
Representing states

- **Size of a problem** usually described in terms of possible **number of states**
  - Tic-Tac-Toe has about 3^9 states ($19{,}683 \approx 2*10^4$)
  - Checkers has about 10^{40} states
  - Rubik's Cube has about 10^{19} states
  - Chess has about 10^{120} states in a typical game
  - Go has $2*10^{170}$
- State space size ≈ solution difficulty

Representing states

- Our estimates were loose upper bounds
- How many **possible, legal** states does tic-tac-toe really have?
- Simple upper bound: nine board cells, each of which can be empty, O or X, so 3^9
- Only 593 states after eliminating
  - impossible states 

  - Rotations and reflections 



Can a Problem space be infinite?

Yes, examples include theorem proving and this simple example from Knuth (1964)

- Starting with the number 4, a sequence of square root, floor, and factorial operations can reach any desired positive integer
- To get to 5 from 4, do

$$\left\lfloor \sqrt{\sqrt{\sqrt{\sqrt{\sqrt{(4!)!}}}}} \right\rfloor = 5.$$

- floor(sqrt (sqrt (sqrt (sqrt (sqrt (fact (fact 4)))))))

Are they infinitely hard to solve?

- No
- But you must be more careful in searching a space that may be infinite
- Some approaches (e.g. breadth first search) may be better than others (e.g., depth first search)

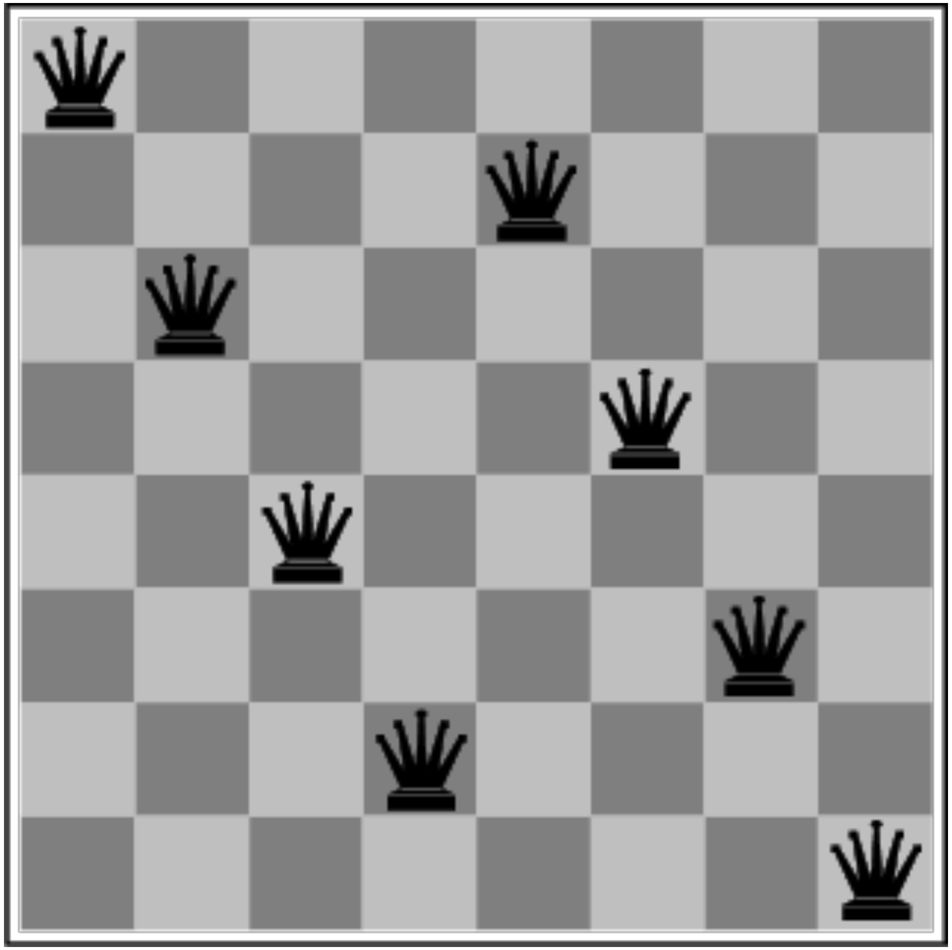
Some example problems

- Toy problems and micro-worlds
  - 8-Puzzle
  - Missionaries and Cannibals
  - Cryptarithmetic
  - Remove 5 Sticks
  - Water Jug Problem
- Real-world problems

The 8-Queens Puzzle

Place eight queens on a chessboard such that no queen attacks any other

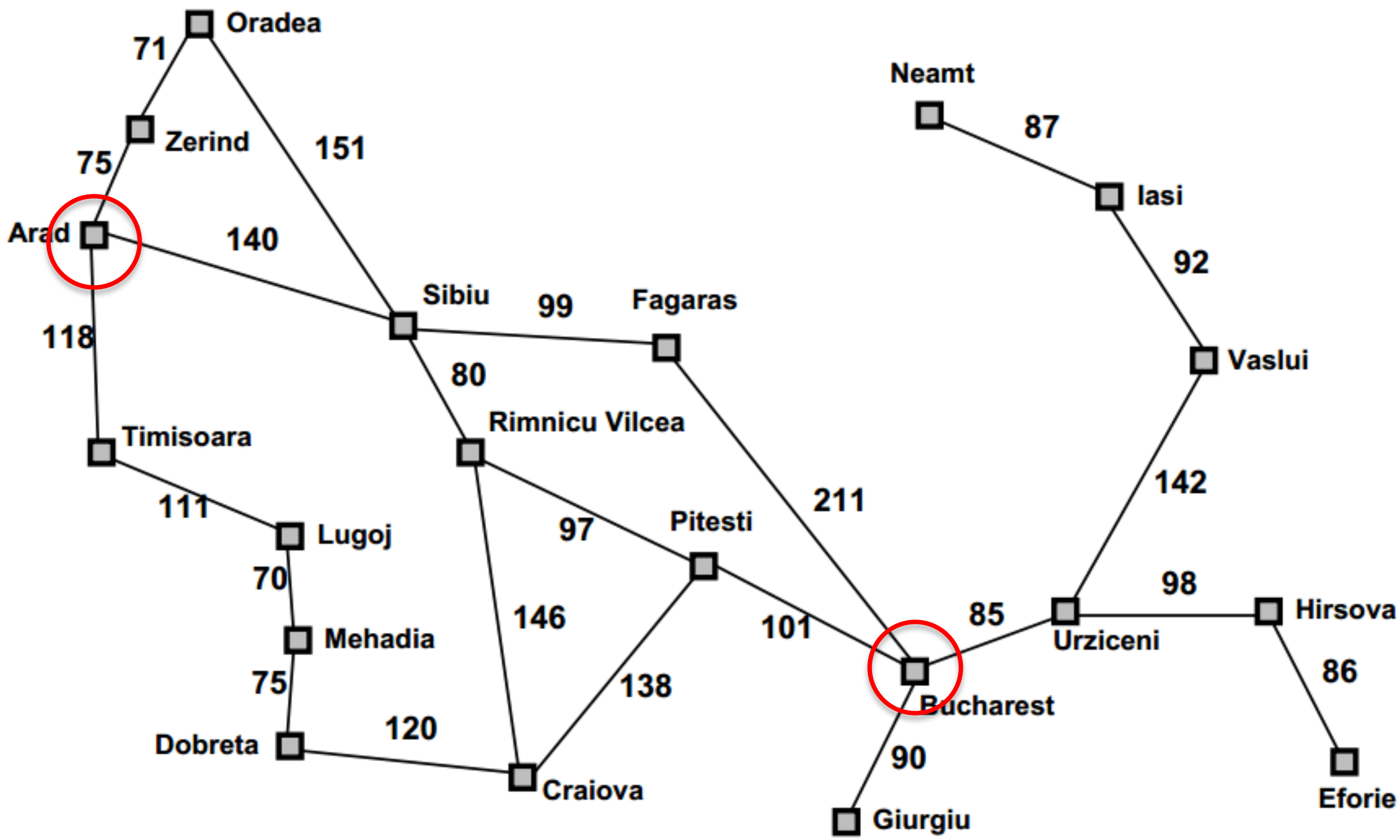
We can generalize the problem to a NxN chessboard



What are the states, goal test, actions?

Route Planning

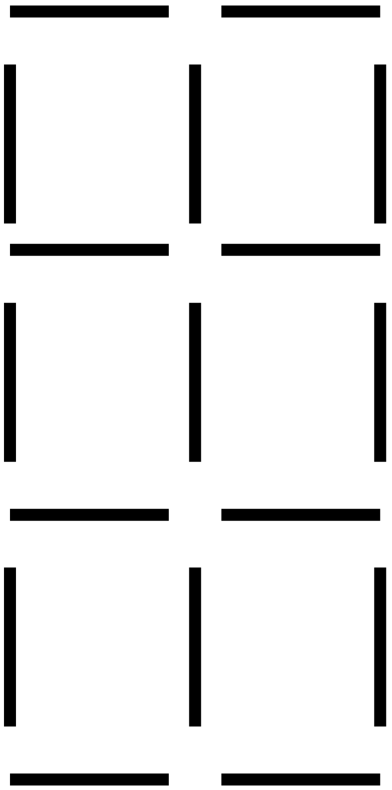
Find a route from Arad to Bucharest



A simplified map of major roads in Romania used in our text

Remove 5 Sticks

Given this configuration of sticks, remove exactly five sticks so that the remaining ones form exactly three squares



Other tasks:

- Remove 4 sticks and leave 4 squares
- Remove 3 sticks and leave 4 squares
- Remove 4 sticks and leave 3 squares

Water Jug Problem

- Two jugs J1 & J2 with capacity C1 & C2
- Initially J1 has W1 water and J2 has W2 water
  - e.g.: full 5 gallon jug and empty 2 gallon jug
- Possible actions:
  - Pour from jug X to jug Y until X empty or Y full
  - Empty jug X onto the floor
- Goal: J1 has G1 water and J2 G2
  - G1 or G2 can be -1 to represent any amount
- E.g.: initially full jugs with capacities 3 and 1 liters, goal is to have 1 liter in each

So…

- How can we represent the states?
- What's an initial state; how to recognize a goal state
- What are the actions; how can we tell which can be done in a given state; what's the resulting state
- How do we search for a solution from an initial state any goal state
- What is a solution, e.g.:
  - The goal state achieved, or
  - The path (i.e., sequence of actions) taking us from the initial state to a goal state?

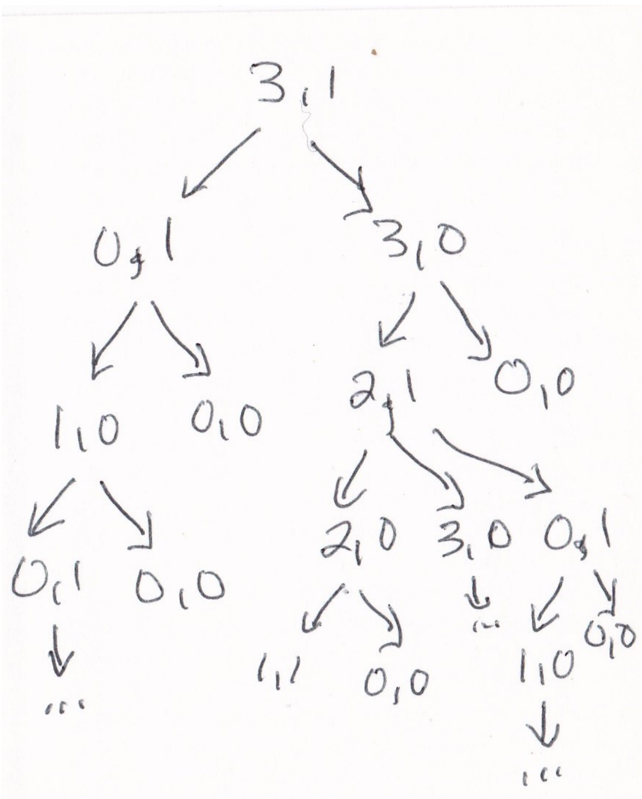
Search in a state space

- Basic idea:
  - Create representation of initial state
  - Try all possible actions & connect states that result
  - Recursively apply process to the new states until we find a solution or dead ends
- We need to keep track of the connections between states and might use a
  - Tree data structure or
  - Graph data structure
- A graph structure is best in general…

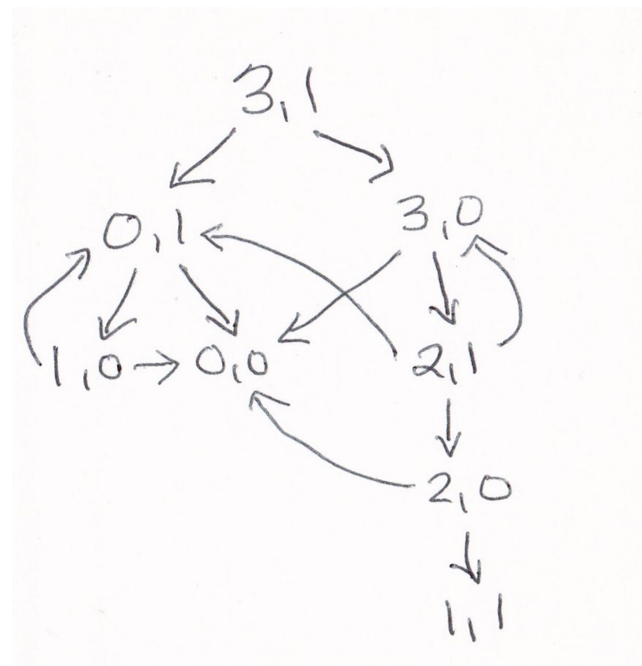
Search in a state space

Consider a water jug problem with a 3-liter and 1-liter jug, an initial state of (3,1) and a goal stage of (1,1)

Tree model of space



Graph model of space



graph model avoids redundancy and loops and is usually preferred

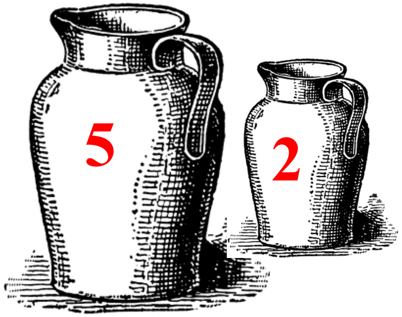
Formalizing state space search

- A state space is a **graph** (V, E) where V is a set of **nodes** and E is a set of **arcs**, and each arc is directed from a node to another node
- **Nodes:** data structures with state description and other info, e.g., node's parent, name of action that generated it from parent, etc.
- **Arcs:** instances of actions, head is a state, tail is the state that results from action, label on arc is action's name or id

Formalizing search in a state space

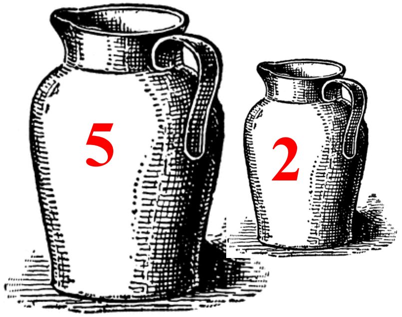
- Each arc has fixed, positive **cost** associated with it corresponding to the action cost
  - Simple case: all costs are 1
- Each node has a set of **successor nodes** corresponding to all legal actions that can be applied at node's state
  - **Expanding** a node = generating its successor nodes and adding them and their associated arcs to the graph
- One or more nodes are marked as **start nodes**
- A **goal test** predicate is applied to a state to determine if its associated node is a goal node

Example: Water Jug Problem



- Two jugs J1 and J2 with capacity C1 and C2
- Initially J1 has W1 water and J2 has W2 water
  - e.g.: a full 5-gallon jug and an empty 2-gallon jug
- Possible actions:
  - Pour from jug X to jug Y until X empty or Y full
  - Empty jug X onto the floor
- Goal: J1 has G1 water and J2 G2
  - G1 or G0 can be -1 to represent any amount

Example: Water Jug Problem



Given full 5-gal. jug and empty 2-gal. jug, fill 2-gal jug with one gallon

- State representation?
  - –General state?
  - –Initial state?
  - –Goal state?
- Possible actions?
  - –Condition?
  - –Resulting state?

Action table

Name	Cond.	Transition	Effect

Example: Water Jug Problem

Given full 5-gal. jug and empty 2-gal. jug, fill 2-gal jug with one gallon

- State = (x,y), where x is water in jug 1; y is water in jug 2
- Initial State = (5,0)
- Goal State = (-1,1), where -1 means any amount

Action table

Name	Cond.	Transition	Effect
dump1	x>0	(x,y)→(0,y)	Empty Jug 1
dump2	y>0	(x,y)→(x,0)	Empty Jug 2
pour_1_2	x>0 & y<C2	(x,y)→(x-D,y+D) *D = min(x,C2-y)*	Pour from Jug 1 to Jug 2
pour_2_1	y>0 & X<C1	(x,y)→(x+D,y-D) *D = min(y,C1-x)*	Pour from Jug 2 to Jug 1

Formalizing search

- **Solution:** sequence of actions associated with a path from a start node to a goal node
- **Solution cost:** sum of the arc costs on the solution path
  - If all arcs have same (unit) cost, then solution cost is length of solution (number of steps)
  - Algorithms generally require that arc costs cannot be negative (why?)

Formalizing search

- **State-space search:** searching through state space for solution by **making explicit** a portion of an **implicit** state-space graph to find a goal node
  - Can't materializing whole space for large problems
  - Initially V={S}, where S is the start node, E={}
  - On **expanding** S, its ***successor nodes*** are generated and added to V and associated ***arcs added*** *to E*
  - Process continues until a goal node is found
- Nodes represent a *partial solution* path (+ cost of partial solution path) from S to the node
  - From a node there may be many possible paths (and thus solutions) with this partial path as a prefix

State-space search algorithm

;; problem describes the start state, operators, goal test, and operator costs
;; queueing-function is a comparator function that ranks two states
;; general-search returns either a goal node or failure

```
function general-search (problem, QUEUEING-FUNCTION)
  nodes = MAKE-QUEUE(MAKE-NODE(problem.INITIAL-STATE))
  loop
     if EMPTY(nodes) then return "failure"
     node = REMOVE-FRONT(nodes)
     if problem.GOAL-TEST(node.STATE) succeeds
        then return node
     nodes = QUEUEING-FUNCTION(nodes, EXPAND(node,
                problem.OPERATORS))
 end
```

;; Note: The goal test is NOT done when nodes are generated
;; Note: This algorithm does not detect loops

Key procedures to be defined

- EXPAND
  - Generate a node's successor nodes, adding them to the graph if not already there
- GOAL-TEST
  - Test if state satisfies all goal conditions
- QUEUEING-FUNCTION
  - Maintain ranked list of nodes that are candidates for expansion
  - Changing definition of the QUEUEING-FUNCTION leads to different search strategies: Which node to expand next

Bookkeeping

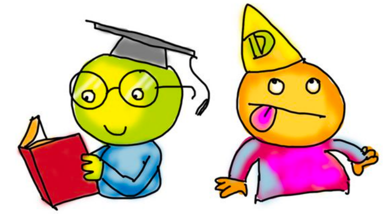
Typical node data structure includes:

- State at this node
- Parent node(s)
- Action(s) applied to get to this node
- Depth of this node (# of actions on shortest known path from initial state)
- Cost of path (sum of action costs on best path from initialstate)

Some issues

- Search process constructs a search tree/graph, where
  - **root** is initial state and
  - **leaf nodes** are nodes
    - not yet expanded (i.e., in list "nodes") or
    - having no successors (i.e., they're *deadends* because no operators were applicable and yet they are not goals)
- Search tree may be infinite due to loops; even graph may be infinite for some problems
- Solution is a *path* or a *node*, depending on problem.
  - E.g., in cryptarithmetic return a node; in 8-puzzle, a path
- Changing definition of the QUEUEING-FUNCTION leads to different search strategies

Informed vs. uninformed search



Uninformed search strategies (blind search)

- Use no information about likely *direction* of a goal
- Methods: breadth-first, depth-first, depth-limited, uniform-cost, depth-first iterative deepening, bidirectional

Informed search strategies (heuristic search)

- Use information about domain to (try to) (usually) head in the general direction of goal node(s)
- Methods: hill climbing, best-first, greedy search, beam search, algorithm A, algorithm A*

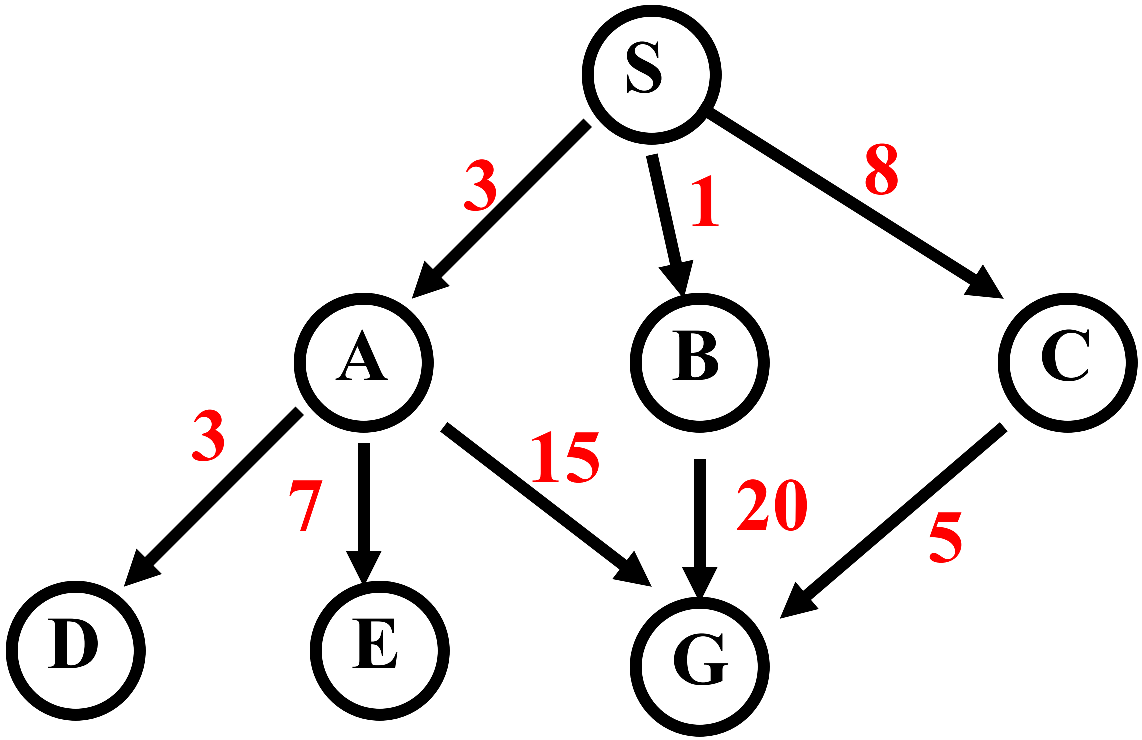
Evaluating search strategies

- **Completeness**
  - Guarantees finding a solution whenever one exists
- **Time complexity** (worst or average case)
  - Usually measured by *number of nodes expanded*
- **Space complexity**
  - Usually measured by maximum size of graph/tree during the search
- **Optimality (aka [Admissibility])**
  - If a solution is found, is it **guaranteed** to be an optimal one, i.e., one with minimum cost

Classic uninformed search methods

- The four classic uninformed search methods
  - Breadth first search (BFS)
  - Depth first search (DFS)
  - Uniform cost search *(generalization of BFS)*
  - Iterative deepening *(blend of DFS and BFS)*
- To which we can add another technique
  - Bi-directional search *(hack on BFS)*

Example of uninformed search strategies

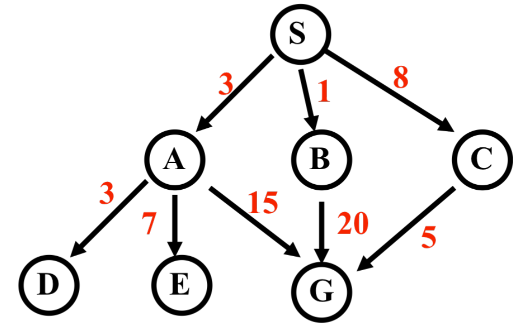


Consider this search space where S is the start node, G is the goal, and numbers are arc costs

assume graph is not known in advance

Breadth-First Search

ignore weights on arcs



Expanded node	Nodes list (aka Fringe)
	{ S^0 }
S^0	{ A^3 B^1 C^8 }
A^3	{ B^1 C^8 D^6 E^{10} G^{18} }
B^1	{ C^8 D^6 E^{10} G^{18} G^{21} }
C^8	{ D^6 E^{10} G^{18} G^{21} G^{13} }
D^6	{ E^{10} G^{18} G^{21} G^{13} }
E^{10}	{ G^{18} G^{21} G^{13} }
G^{18}	{ G^{21} G^{13} }

FIFO (queue)

Notation

G^{18}

G is node; 18 is cost of shortest known path from S

- Typically don't check if node is goal until we expand it (why?)
- Solution path found is S A G , cost 18
- # nodes expanded (including goal node) = 7

Breadth-First Search (BFS)

- **Long time to find solutions** with many steps: we must look at all shorter length possibilities first
  - Complete tree of depth d where nodes have b children has $1+b+b^2+...+b^d = (b^{(d+1)}-1)/(b-1)$ nodes = $\mathbf{0(b^d)}$
  - Tree with depth 12 & branching 10 > trillion nodes
  - If BFS expands 1000 nodes/sec and nodes uses 100 bytes, can take 35 years & uses 111TB of memory!

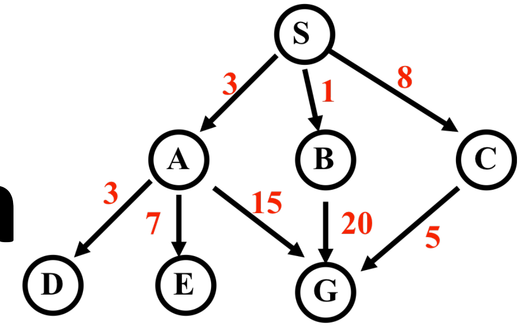
+ Always finds solution if one exists

+ Solution found is optimal

Breadth-First Search

- Enqueue nodes in **FIFO** (first-in, first-out) order
- **Complete**
- **Optimal** (i.e., admissible) finds shorted path, which is optimal if all operators have same cost
- **Exponential time and space complexity**, $O(b^d)$, where d is depth of solution; b is branching factor (i.e., # of children)
- **Long time to find long solutions** since we explore all shorter length possibilities first

Depth-First Search



Expanded node	Nodes list (aka fringe)
	{ S^0 }
S^0	{ A^3 B^1 C^8 }
A^3	{ D^6 E^{10} G^{18} B^1 C^8 }
D^6	{ E^{10} G^{18} B^1 C^8 }
E^{10}	{ G^{18} B^1 C^8 }
G^{18}	{ B^1 C^8 }

LIFO (stack)

Solution path found is S A G, cost 18

Number of nodes expanded (including goal node) = 5

Depth-First (DFS)

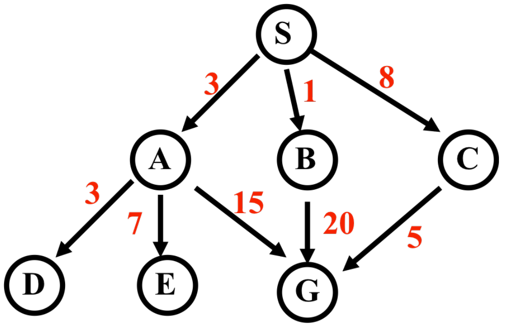
- Enqueue nodes on nodes in **LIFO** (last-in, first-out) order, i.e., use stack data structure to order nodes
- **May not terminate** w/o *depth bound*, i.e., ending search below fixed depth D (depth-limited search)
- **Not complete** (with or w/o cycle detection, with or w/o a cutoff depth)
- **Exponential time**, $O(b^d)$, but **linear space**, $O(bd)$
- Can find **long solutions quickly** if lucky (and **short solutions slowly** if unlucky!)
- On reaching deadend, can only back up one level at a time even if problem occurs because of a bad choice at top of tree

Uniform-Cost Search (UCS)

- Enqueue nodes by **path cost**. i.e., let g(n) = cost of path from *start* to current node *n*. Sort nodes by increasing value of g(n).
- Aka *Dijkstra's Algorithm and* similar to *Branch and Bound Algorithm* from operations research
- **Complete (*)**
- **Optimal/Admissible** (*)

  Depends on goal test being applied *when node is removed from nodes list*, not when its parent node is expanded & node first generated
- **Exponential time and space complexity**, O(b^d)

Uniform-Cost Search



Expanded node	Nodes list
	$\{ S^0 \}$
S^0	$\{ B^1\ A^3\ C^8 \}$
B^1	$\{ A^3\ C^8\ G^{21} \}$
A^3	$\{ D^6\ C^8\ E^{10}\ G^{18}\ G^{21} \}$
D^6	$\{ C^8\ E^{10}\ G^{18}\ G^{21} \}$
C^8	$\{ E^{10}\ G^{13}\ G^{18}\ G^{21} \}$
E^{10}	$\{ G^{13}\ G^{18}\ G^{21} \}$
G^{13}	$\{ G^{18}\ G^{21} \}$

priority queue

Solution path found is S C G, cost 13

Number of nodes expanded (including goal node) = 7

Depth-First Iterative Deepening (DFID)

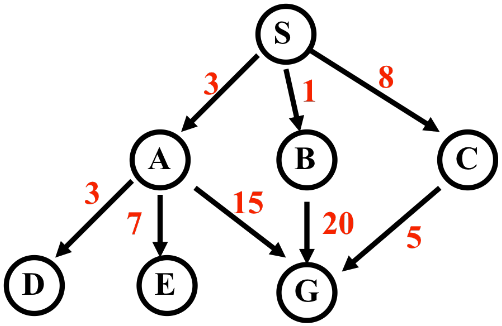
- Do DFS to depth 0, then (if no solution) DFS to depth 1, etc.
- Often used with a tree search
- **Complete**
- **Optimal/Admissible** if all operators have unit cost, else finds shortest solution (like BFS)
- Time complexity a bit worse than BFS or DFS

  Nodes near top of search tree generated many times, but since almost all nodes are near tree bottom, worst case time complexity still exponential, $O(b^d)$

Depth-First Iterative Deepening (DFID)

- If branching factor is b and solution is at depth d, then nodes at depth d are generated once, nodes at depth d-1 are generated twice, etc.
  - Hence $b^d + 2b^{(d-1)} + ... + db <= b^d / (1 - 1/b)^2 = O(b^d)$.
  - If b=4, worst case is $1.78 * 4^d$, i.e., 78% more nodes searched than exist at depth d (in worst case)
- **Linear space complexity**, O(bd), like DFS
- Has advantages of BFS (completeness) and DFS (i.e., limited space, finds longer paths quickly)
- Preferred for **large state spaces** where **solution depth is unknown**

How they perform



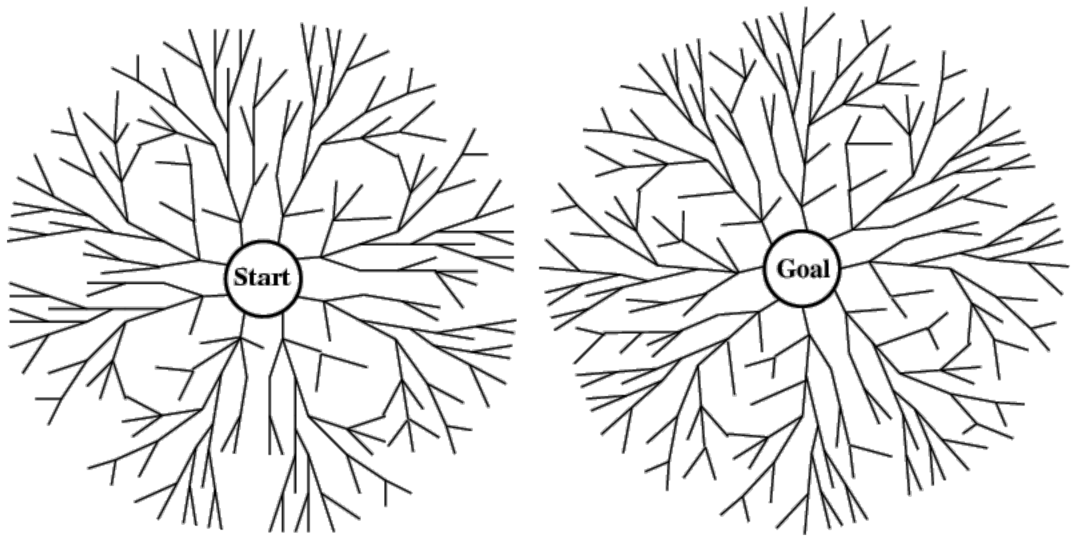
- **Depth-First Search:**
  - 4 Expanded nodes: S A D E G
  - Solution found: S A G (cost 18)
- **Breadth-First Search**:
  - 7 Expanded nodes: S A B C D E G
  - Solution found: S A G (cost 18)
- **Uniform-Cost Search**:
  - 7 Expanded nodes: S A D B C E G
  - Solution found: S C G (cost 13)

  Only uninformed search that worries about costs
- **Iterative-Deepening Search**:
  - 10 nodes expanded: S S A B C S A D E G
  - Solution found: S A G (cost 18)

Searching Backward from Goal

- Usually a successor function is reversible
  - i.e., can generate a node’s predecessors in graph
- If we know a single goal (rather than a goal’s properties), we could search backward to the initial state
- It might be more efficient
  - Depends on whether the graph fans in or out

Bi-directional search



- Alternate searching from the start state toward the goal and from the goal state toward the start
- Stop when the frontiers intersect
- Works well only when there are unique start & goal states
- Requires ability to generate "predecessor" states
- Can (sometimes) lead to finding a solution more quickly

Comparing Search Strategies

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Time	b^d	b^d	b^m	b^l	b^d	$b^{d/2}$
Space	b^d	b^d	bm	bl	bd	$b^{d/2}$
Optimal?	Yes	Yes	No	No	Yes	Yes
Complete?	Yes	Yes	No	Yes, if $l \geq d$	Yes	Yes

Summary

- Search in a problem space is at the heart of many AI systems
- Formalizing the search in terms of states, actions, and goals is key
- The simple “uninformed” algorithms we examined can be augmented to heuristics to improve them in various ways
- But for some problems, a simple algorithm is best